UNIT - I

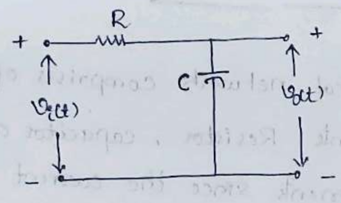
Linear Wave Shaping

Any electrical network comprises of both active and passive elements. Resistor, capacitor and inductor are linear circuit elements since the current through them is proportional to the voltage i.e there is a linear relationship between the applied voltage and the resulting current. A network comprising of these linear elements is termed as "linear network".

When a sinusoidal voltage is applied to a linear circuit, the output voltage is also sinusoidal in nature. The amplitude of the input signal may change and there may be phase displacement between the output voltage and the input signal. but there is no distortion of waveshape. No other periodic waveform [step, ramp, pulse, square wave and exponential] preserves its shape precisely when transmitted through a linear network.

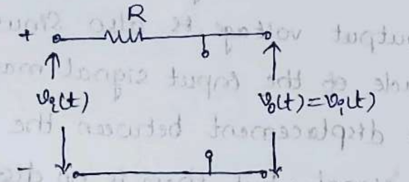
"The process where by the form of a non-sinusoidal signal is altered by transmission through a linear network is called linear wave shaping".

The Low-pass RC Circuit :

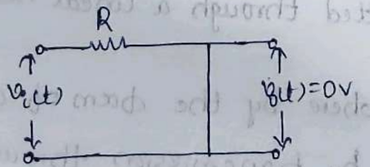


A lowpass circuit is a circuit, which transmits only low-frequency signals and attenuates or stops high-frequency signals.

(i) At $f=0$, $X_C = \frac{1}{2\pi f C} = \infty$ Capacitor open circuited

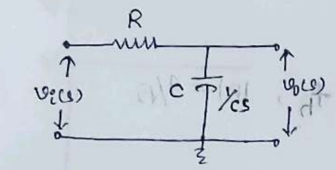


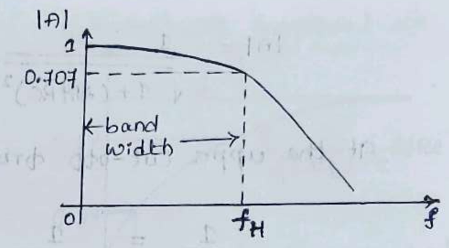
(ii) At $f=\infty$, $X_C = \frac{1}{2\pi f C} = 0$ capacitor short circuited.



From the two conditions when frequency is low the output is following the input. when frequency is High the output is zero, so the lowpass filter allows only low frequencies and attenuates high frequencies.

Sinusoidal Input :





The gain versus frequency curve of a Low-pass circuit excited by a sinusoidal input is shown in figure. This is obtained by keeping the amplitude of the input sinusoidal signal constant and varying its frequency and noting the output at each frequency. At low frequencies the output is equal to the input and hence the gain is unity. As the frequency increases, the output decrease and hence gain decreases. The frequency at which the gain is $1/\sqrt{2}$ (0.707) of its maximum value is called the cut-off frequency. For a low-pass filter there is no lower-cutoff frequency, it is zero itself. The upper cut-off frequency is the frequency at which the gain is $1/\sqrt{2}$ i.e 70.7% of its maximum value.

$$A = \frac{v_o(s)}{v_i(s)}$$

$$v_o(s) = \frac{v_i(s)\,(1/sC)}{R + 1/sC}$$

$$\frac{v_o(s)}{v_i(s)} = \frac{1}{1+sCR} = \frac{1}{1+j\omega RC} = \frac{1}{1+j2\pi fRC}$$

$$|A| = \frac{1}{\sqrt{1+(2\pi fRC)^2}}$$

At the upper cut-off frequency f_h, $|A| = 1/\sqrt{2}$

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1+(2\pi f_h RC)^2}}$$

Squaring on both sides and equating the denominators

$$2 = 1+(2\pi f_h RC)^2 \Rightarrow 2\pi f_h RC = 1$$

The upper cut-off frequency $\boxed{f_h = 1/2\pi RC}$

$$\text{So } A = \frac{1}{1+j2\pi fRC} = \frac{1}{1+j(f/(\frac{1}{2\pi RC}))}$$

$$A = \frac{1}{1+j(f/f_h)}$$

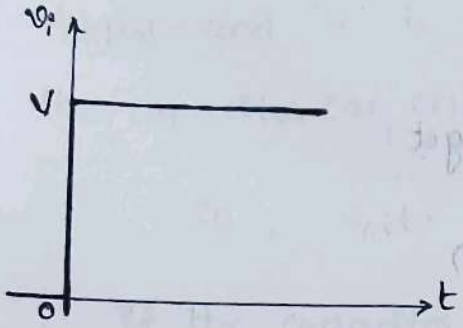
$$|A| = \frac{1}{\sqrt{1+(f/f_h)^2}}$$

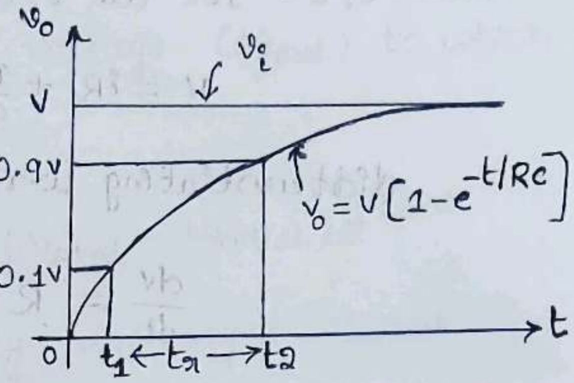
phase angle 'θ' by which the output leads the input is given by

$$\theta = \frac{\tan^{-1}(\frac{0}{1})}{\tan^{-1}(f/f_h)} = \tan^{-1}(0) - \tan^{-1}(f/f_h)$$

$$\boxed{\theta = -\tan^{-1}(f/f_h)}$$

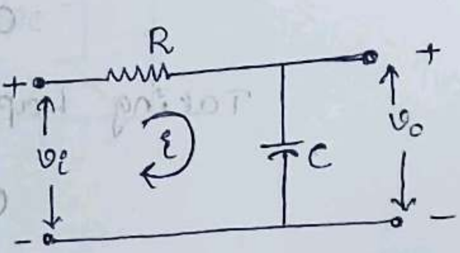
Step-voltage Input :





$v_i = 0$ for $t<0$

$= V$ for $t>0$



The function be a step voltage 'V', due to the applied voltage a current 'i' results. It is evident that at $t=(0^-)$, $v_i=0$ and $v_o=0$

At $t=(0^+)$ also $v_i=0$ and $v_o=0$

For $t>0$, a current flows and the capacitor charges. As charging progresses more and more voltage develops across 'C' and the current 'i' progressively diminishes and eventually it becomes zero.

Applying KVL to the RC circuit for $t>0$

$$v_i = v_R + v_c$$

where $v_R = iR$; $v_c = v_o = \frac{1}{C}\int i\,dt$

$$v_i = iR + \frac{1}{C}\int i\,dt$$

But $v_i = V$ for all $t > 0$

$$V = iR + \frac{1}{C}\int i\,dt$$

differentiating w.r.t 't', we get

$$\frac{dV}{dt} = R\frac{di(t)}{dt} + \frac{1}{C}i(t)$$

$\frac{dV}{dt} = 0$, since 'V' is of constant magnitude

$$0 = R\frac{di(t)}{dt} + \frac{i(t)}{C}$$

Taking Laplace transform on both sides

$$0 = R\left[sI(s) - I(0^+)\right] + \frac{1}{C}I(s)$$

$$I(0^+) = I(s)\left[s + 1/RC\right]$$

The initial current $I(0^+)$ is given by

$$I(0^+) = \frac{V - V'}{R}$$

$$I(s) = \frac{I(0^+)}{(s + 1/RC)} = \frac{V - V'}{R\left[s + 1/RC\right]}$$

and $v_0(s) = v_i(s) - V_R(s) = v_i(s) - I(s)R$

$$v_0(s) = \frac{V}{s} - \frac{(V - V')\cancel{R}}{\cancel{R}\left[s + 1/RC\right]}$$

Taking inverse Laplace transform on both sides

$$v_0(t) = V - (V - V')e^{-t/RC}$$

where v' is the initial voltage across the capacitor $V_{initial}$ and 'V' is the final voltage (V_{final}) to which the capacitor can charge.

So, $v_o(t) = V_{final} - (V_{final} - V_{initial}) e^{-t/RC}$

If the capacitor is initially uncharged $V_{initial} = 0$

$$v_o(t) = V_{final} - (V_{final}) e^{-t/RC}$$

$$\boxed{v_o(t) = V\left[1 - e^{-t/RC}\right]}$$

Rise time:

When a step signal is applied, the rise time 't_r' is defined as the time taken by the output voltage waveform to rise from 10% to 90% of its final value. It gives an indication of how fast the circuit can respond to a discontinuity in voltage.

The o/p voltage at any instant of time is given by

$$v_o(t) = V\left[1 - e^{-t/RC}\right]$$

At $t = t_1$, $v_o = 0.1V$ [i.e 10% of V]

$$0.1V = V\left[1 - e^{-t_1/RC}\right]$$

$$e^{-t_1/RC} = 0.9 \quad \text{(or)} \quad e^{t_1/RC} = 1/0.9 = 1.11$$

$$t_1 = RC \ln(1.11) = 0.1RC$$

At $t = t_2$, $v_o(t) = 0.9V$ [i.e 90% of V]

$$0.9V = V\left[1 - e^{-t_2/RC}\right]$$

$$e^{-t_2/RC} = 0.1 \quad \text{(or)} \quad e^{t_2/RC} = \frac{1}{0.1} = 10$$

$$t_2 = RC \ln(10) = 2.3RC$$

Rise time $t_r = t_2 - t_1 = 2.3RC - 0.1RC$

$$\boxed{t_r = 2.2RC}$$

This indicates that the rise time 't_r' is proportional to the time constant 'RC' of the circuit. The larger the time constant, the slower the capacitor charges and smaller the time constant, faster the capacitor charges.

Relation between rise time and upper 3-db frequency:

We know that upper 3-db of LPF is $f_h = 1/2\pi RC$

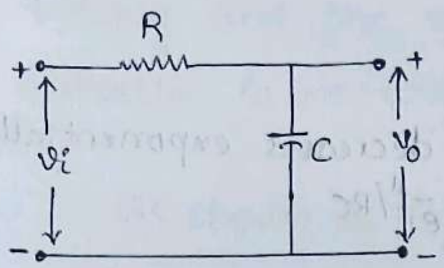
(or) $RC = 1/2\pi f_h$

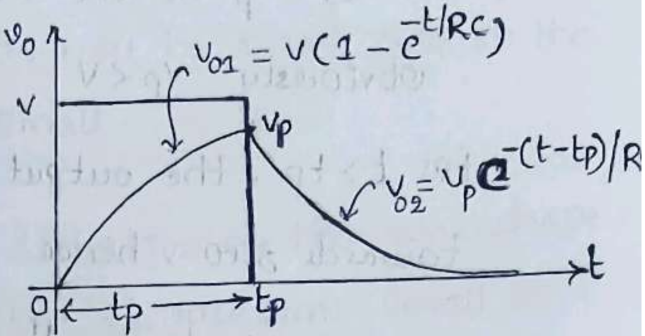
Rise time $t_r = 2.2RC = \dfrac{2.2}{2\pi f_h} = \dfrac{0.35}{f_h} = \dfrac{0.35}{BW}$

$$BW = f_h - f_L$$

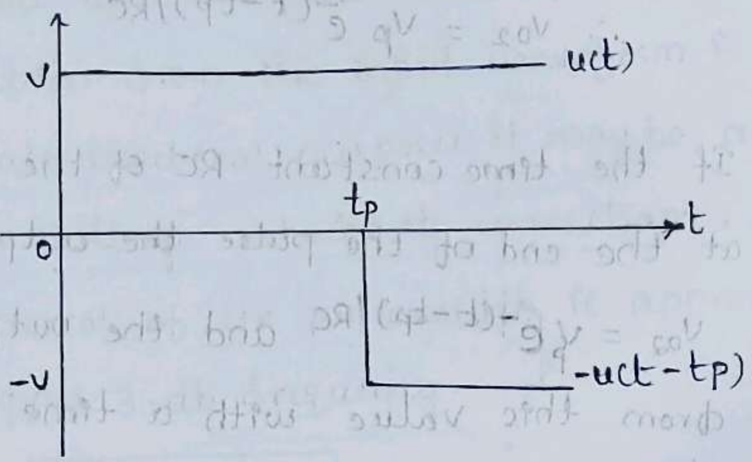
Thus the rise time is inversely proportional to the upper 3-db frequency. The time constant ($\tau = RC$) of a circuit is defined as time taken by the output to rise to 63.2% of the amplitude of the input step.

Pulse Input:





(a) $RC >> t_p$



(b) pulse interms of steps

The pulse is formed by positive step followed by a decayed negative step. So, the response of Low-pass RC circuit to a pulse for times less than the pulse width t_p is the same as that for step input and is given by $v_o(t) = V[1-e^{-t/RC}]$, hence the output voltage increases exponentially from 0 to v_p at the end of the pulse.

At $t = t_p$, the input voltage drops to zero, and hence th output decreases exponentially and becomes zero at $t = \infty$

while the output voltage is exponentially increasing,

$$v_{o1} = V[1-e^{-t/RC}]$$

where V = amplitude of input pulse

Let $v_{o2} = v_p$ at the end of the pulse i.e at $t = t_p$

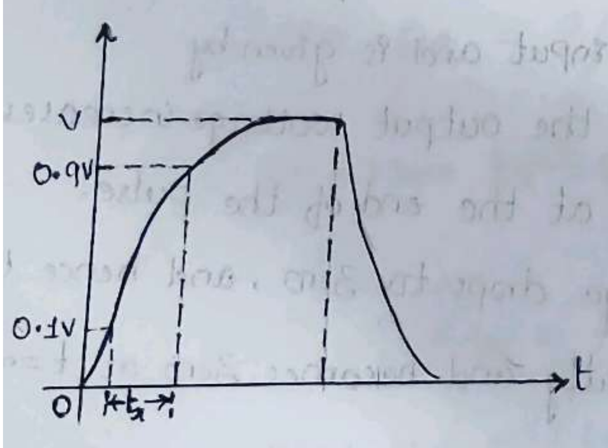
obviously $v_p < V$

for $t > t_p$, the output will be decreases exponentially towards zero, hence $v_{o2} = v_p e^{-t'/RC}$

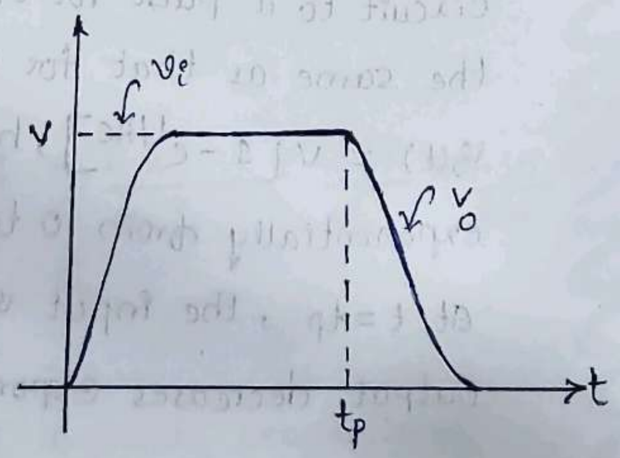
where $t' = t - t_p$

$$\therefore \; v_{o2} = v_p e^{-(t-t_p)/RC}$$

If the time constant RC of the circuit is very large, at the end of the pulse the output voltage will be $v_{o2} = v_p e^{-(t-t_p)/RC}$ and the output decreases to zero from this value with a time constant RC. The pulse waveform is distorted after the 'v_p' and the output will always extend beyond the pulse width t_p, because whatever charge has accumulated across the capacitor 'C' during the pulse cannot changes instantaneously.



(c) $RC < t_p$



(d) $RC << t_p$

As shown in fig(c) if time constant RC of the circuit is very small, the capacitor charges and discharges very quickly and the rise time t_r will be small and so the distortion in the waveshape small.

As shown in fig(d) the distortion of the waveshape can be minimized by making the 't_r' rise time small in comparison with the pulse width 't_p'. In the figure (d) the deviation from the input waveform is quite small and for all practical purposes it may be assumed that the pulse voltage retains its waveshape, provided that the reciprocal of the pulse width is approximately equal to the upper 3-db frequency.

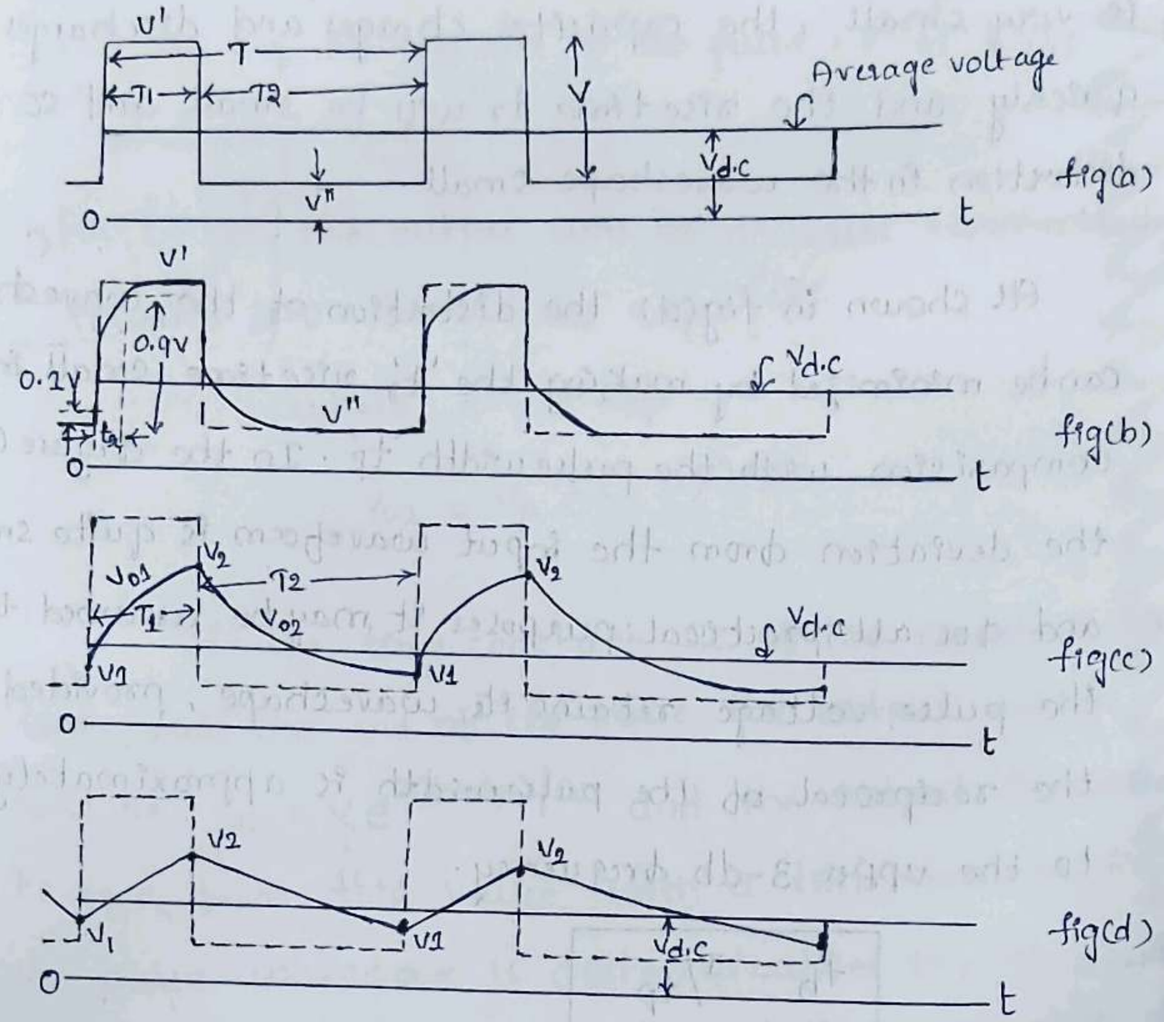
$$\boxed{f_h = 1/t_p}$$

with $f_h = 1/t_p$, we have $t_r = \frac{0.35}{f_h} = 0.35\, t_p$

$$\boxed{t_r = 0.35\, t_p}$$

"A pulse shape can be preserved if upper 3-db frequency is approximately equal to the reciprocal of the pulse width".

Square Wave Input :



(a) square wave Input (b) o/p for $RC << T$ (c) o/p for $RC \approx T$ and (d) o/p for $RC >> T$

A square wave is a periodic waveform which maintains itself at one constant level v' with respect to ground for a time $T1$ and then charges abruptly to another level v'' and remains constant at that level for time $T2$, and repeats itself at regular intervals of $T = T1 + T2$.

The shape of the output waveform for a square wave input depends on the time constant of the circuit.

If the time constant is very small the rise time will also be small and a reasonable reproduction of the Input may be obtained.

If the time constant RC of the circuit is small compared to the period of the input waveform the output will be reasonable reproduction of the Input waveform. In this case, the wave is preserved. If the time constant is comparable with the period of the input square wave, the output will be as shown in fig(c). The output rises and falls exponentially. If the time constant is very large compared to the period of the Input waveform, the o/p consists of exponential sections which are essentially linear as shown in fig(d)

The equation of the rising portion is determined by the fact that it must be an exponential of rising time constant RC, and the voltage would rise to V'. If V1 is the initial value of the output voltage

$$V_{01} = V' + [V_1 - V'] e^{-t/RC} \quad \text{——— (1)}$$

Similarly the equation of the falling portion is

$$V_{02} = V'' + (V_2 - V'') e^{-(t-T_1)/RC} \quad \text{——— (2)}$$

At $t = T_1$, $V_{01} = V_2$ and at $t = T_1 + T_2$, $V_{02} = V_1$

the two resulting equations can be solved for the two unknowns V1 and V2.

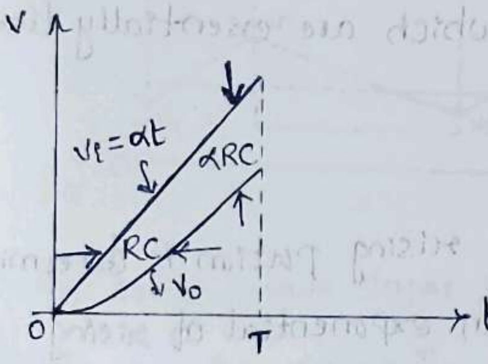
Since the average voltage across 'R' is zero, then the dc voltage at the O/P is the same as that of the input. This average value is indicated as V_{dc} in all the waveforms.

Consider a symmetrical square wave with zero average value, so that $T1 = T2 = T/2$ and $V' = -V'' = V/2$ and $V1 = -V2$, we find that

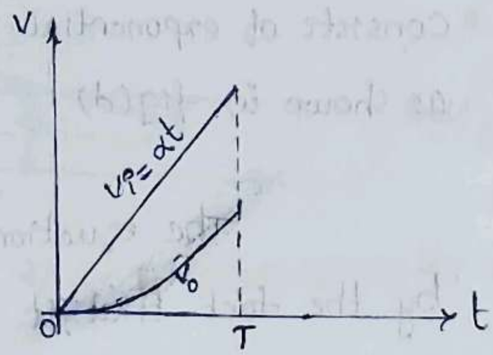
$$V1 = \frac{V}{2}\left[\frac{e^{2x}-1}{e^{2x}+1}\right] = \frac{V}{2}\tanh x = \frac{V}{2}\left[\frac{1-e^{-T/2RC}}{1+e^{-T/2RC}}\right]$$

where T = time period ∴ $x = T/4RC$

Ramp Input :



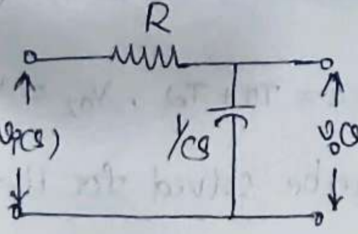
(a) $RC/T << 1$ [$RC << T$]



(b) $RC/T >> 1$ [$RC >> T$]

When a Lowpass RC circuit is excited by a ramp input i.e $v_i(t) = \alpha t$ where 'α' is slope of the ram

$$V_i(s) = \frac{\alpha}{s^2} \qquad L(t^n) = \frac{1}{s^{n+1}}$$



$$V_o(s) = V_i(s) \cdot \frac{1/sc}{R + 1/sc}$$

$$V_o(s) = \frac{\alpha}{s^2} \cdot \frac{1}{1+sCR} = \frac{\alpha}{RC} \cdot \frac{1}{s^2(s+1/RC)}$$

$$= \frac{\alpha}{RC}\left[\frac{-(RC)^2}{s} + \frac{RC}{s^2} + \frac{(RC)^2}{s+1/RC}\right] \quad \left\{\begin{array}{l}\text{By applying} \\ \text{partial fractions}\end{array}\right\}$$

i.e $$V_o(s) = \frac{-\alpha RC}{s} + \frac{\alpha}{s^2} + \frac{\alpha RC}{s+1/RC}$$

Taking the inverse laplace transform on both sides

$$V_o(t) = -\alpha RC + \alpha t + \alpha RC\, e^{-t/RC}$$

$$= \alpha(t-RC) + \alpha RC\, e^{-t/RC}$$

If the time constant RC is very small $e^{-t/RC} = 0$

$$V_o(t) = \alpha(t-RC)$$

when the time constant is very small relative to the total ramp time T, the ramp will be transmitted with minimum distortion. The o/p follows the Input but is delayed by one time constant RC from the input as shown in fig(a). If the time constant is large as compared with the sweep duration i.e $RC/T >> 1$ the o/p will be highly distorted as shown in fig(b).

Expanding $e^{-t/RC}$ into an indefinite series interms of t/RC in the above equation for $V_o(t)$

$$V_o(t) = \alpha(t-RC) + \alpha RC \cdot e^{-t/RC}$$

$$v_o(t) = \alpha(t-RC) + \alpha RC\left[1 - \frac{t}{RC} + \left(\frac{t}{RC}\right)^2 \cdot \frac{1}{2!} - \left(\frac{t}{RC}\right)^3 \cdot \frac{1}{3!} + \cdots\right]$$

$$= \alpha t - \alpha RC + \alpha RC - \alpha t + \frac{\alpha t^2}{2RC} - \cdots$$

$$= \frac{\alpha t^2}{2RC} \approx \frac{\alpha}{RC}\left(\frac{t^2}{2}\right)$$

$$\boxed{v_o(t) = \frac{\alpha}{RC}\left(\frac{t^2}{2}\right)}$$

This shows that a quadratic response is obtained for a linear input and hence the circuit acts as an integrator for $RC/T >> 1$

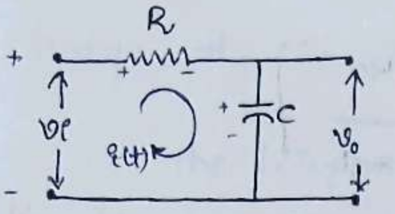
The transmission error e_t for a ramp input is defined as the difference between the input and output divided by the input at the end of the ramp i.e at $t=T$ for $RC/T << 1$

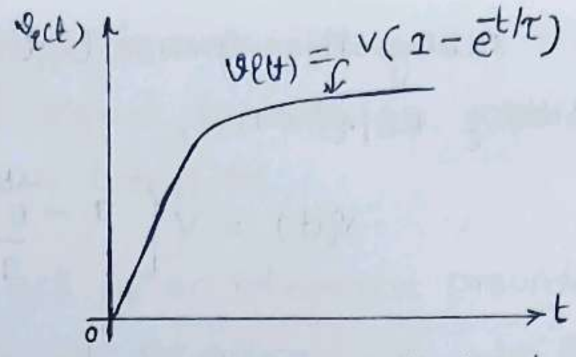
$$e_t = \left.\frac{\alpha t - (\alpha t - \alpha RC)}{\alpha t}\right|_{t=T}$$

$$= \frac{\alpha RC}{\alpha T} = \frac{RC}{T} = \frac{1}{2\pi f_h T}$$

where 'f_h' is the upper 3-db frequency.

Exponential Input :





Exponential Input

for the lowpass RC circuit exponential input applied as shown in figure $v_i(t) = V[1-e^{-t/\tau}]$, where '$\tau$' is the time constant of the input waveform.

Applying KVL around the loop,

$$v_i(t) = i(t)R + v_o(t) = i(t)R + v_o(t)$$

$$v_i(t) = RC\frac{dv_o(t)}{dt} + v_o(t) \qquad \left\{\because i(t) = C\frac{dv_o(t)}{dt}\right\}$$

$$V(1-e^{-t/\tau}) = RC\frac{dv_o(t)}{dt} + v_o(t)$$

Taking the Laplace transform on bothsides and neglecting the initial conditions

$$\frac{V}{S} - \frac{V}{S+1/\tau} = RCSV_o(s) + V_o(s) \qquad \left[L(e^{-at}) = \frac{1}{S+a}\right]$$

i.e $$V\left[\frac{1/\tau}{S(S+1/\tau)}\right] = V_o(s)\left[1+RCS\right] = RCV_o(s)\left[S+1/RC\right]$$

$$V_o(s) = \frac{V}{RC\tau}\left[\frac{1}{S(S+1/\tau)(S+1/RC)}\right]$$

$$= V\left[\frac{1}{S} - \frac{1}{(1-\frac{RC}{\tau})(S+1/\tau)} + \frac{1}{(\frac{\tau}{RC}-1)(S+1/RC)}\right]$$

Taking the inverse laplace transform on both sides and letting $RC/\tau = n$,

$$v_o(t) = V\left[1 - \frac{e^{-t/\tau}}{1-n} + \frac{e^{-t/RC}}{\frac{1}{n}-1}\right]$$

If $t/\tau = x$, then $v_o(t) = V\left[1 - \frac{e^{-x}}{1-n} + \frac{n e^{-x/n}}{1-n}\right]$ if $n \neq 1$

and $v_o(t) = 1-(1+x)e^{-x}$, if $n=1$

These are the expressions for the voltage across the Capacitor of a low-pass RC excited by an exponential input of rise time $t_{r1} = 2.2\tau$

If an exponential of rise time t_{r1} is passed through a lowpass circuit with rise time t_{r2}, the rise time of the output waveform t_r will be given by a relation $t_r = 1.05\sqrt{t_{r1}^2 + t_{r2}^2}$

The Low-pass RC circuit as an Integrator

If the time constant of lowpass RC circuit is very Small, the capacitor charges very slowly and almost all the input voltage appears across the resistor for small values of time. Then the current in the circuit is $v_i(t)/R$ and the output signal across 'C' is

$$v_o(t) = \frac{1}{C}\int i(t)\,dt = \frac{1}{C}\int \frac{v_i(t)}{R}\,dt$$

$$\boxed{v_o(t) = \frac{1}{RC}\int v_i(t)\,dt}$$

hence the output is the integral of the input.

If the time constant is large in comparision with the time required for the input signal to make an appreciable change, the circuit acts as an integrator

The lowpass circuit acts as an integrator provided the time constant of the circuit $RC > 15T$, where 'T' is the period of the input sine wave. and the input sine wave will be shifted by at least 89.4° (Instead of ideal 90° shift required for integration) when it is transmitted through the network.

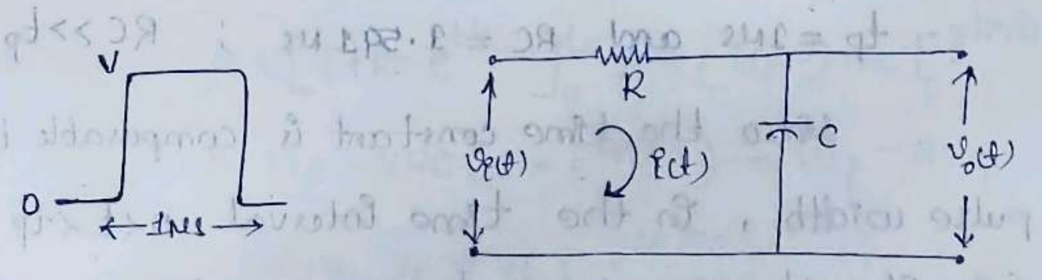
Q: An ideal 1μs pulse is fed to an amplifier. calculate and plot the output waveform under the following conditions
(a) 10MHz (b) 1 MHz (c) 0.1 MHz [upper 3db frequencies]

(a) The upper 3db frequency $f_h = 10MHz$

$$f_h = \frac{1}{2\pi RC} \Rightarrow RC = \frac{1}{2\pi f_h} = \frac{1}{2\pi \times 10 \times 10^6}$$

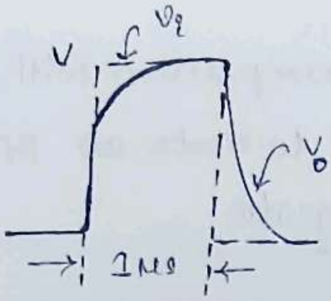
$$RC = 0.0159\ \mu s$$

Since in the circuit the upper 3-db frequency is given, so the ckt acts as lowpass filter.



Since $t_p = 1\mu s$ and $RC = 0.0159\ \mu s$, $RC << t_p$

Since the time constant is very small in comparision with the pulse width, the capacitor 'c' charges rapidly with a rise time.



$t_r = 2.2 \times 0.0159$

$= 0.035\,\mu s$

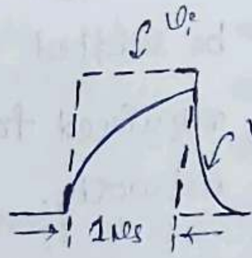
The output voltage v_o is given by

$v_o = V(1 - e^{-t/RC})$

At $t = t_p$, $v_o = V(1 - e^{-1/0.0159}) \approx V$

(b) when $f_h = 1\,MHz$

$RC = 1/2\pi f_h = \frac{1}{2\pi \times 10^6} = 0.1591\,\mu s$



$t_p = 1\,\mu s$ and $RC = 0.1591\,\mu s$

Since, $RC < t_p$ the capacitor charges fast

the rise time $t_r = 2.2RC = 2.2 \times 0.1591 = 0.35\,\mu s$

The o/p voltage is given by $v_o = V(1 - e^{-t/RC})$

At $t = t_p$, $v_o = V(1 - e^{-t_p/RC}) = V(1 - e^{-1/0.1591})$

$v_o = 0.998\,V$

(c) when $f_h = 0.1\,MHz$

$RC = 1/2\pi f_h = \frac{1}{2\pi \times 0.1 \times 10^6} = 1.591\,\mu s$

$t_p = 1\,\mu s$ and $RC = 1.591\,\mu s$; $RC >> t_p$

Since the time constant is comparable to the pulse width, in the time interval $0 < t < t_p$, the capacitor charges exponentially according to the equation

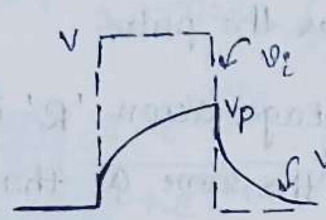
$v_o = V[1 - e^{-t/RC}]$

At $t = t_p$, the o/p voltage $v_o = V_p$

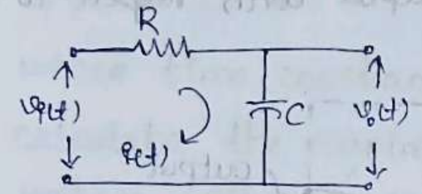
$$v_p = V(1-e^{-t_p/RC}) = V(1-e^{-1/1.591}) = 0.4668V$$

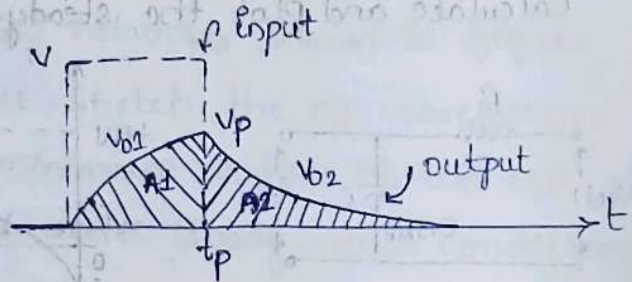
for $t>t_p$, v_o decreases according to the equation

$$v_o = v_p\, e^{-(t-t_p)/RC} = 0.4668\,V\, e^{-(t-1)/1.59}$$



Q8 A pulse is applied to a lowpass RC circuit. prove by direct integration that the area under the pulse is the same as the area under the output waveform across the capacitor. Explain the result physically.





during the pulse duration $0<t<t_p$

Area under the output waveform is

$$A_1 = \int_0^{t_p} v_1(t)\,dt = \int_0^{t_p} V(1-e^{-t/RC})\,dt$$

$$= V\left[t+RC\,e^{-t/RC}\right]_0^{t_p} = V\left[(t_p-0)+RC\left[e^{-t_p/RC}-1\right]\right]$$

$$= Vt_p - VRC(1-e^{-t_p/RC}) = Vt_p - RCV_p$$

$$\text{where } V_p = V(1-e^{-t_p/RC})$$

for $t>t_p$ the area under the o/p waveform is

$$A_2 = \int_{t_p}^{\infty} v_2(t)\,dt = \int_{t_p}^{\infty} V_p\, e^{-(t-t_p)/RC}\,dt$$

$$= (+V_p)\left[\frac{e^{-(t-t_p)/RC}}{(-1/RC)}\right]_{t_p}^{\infty}$$ [capacitor discharges from V_p]

$= (+V_p)(RC) V_p$

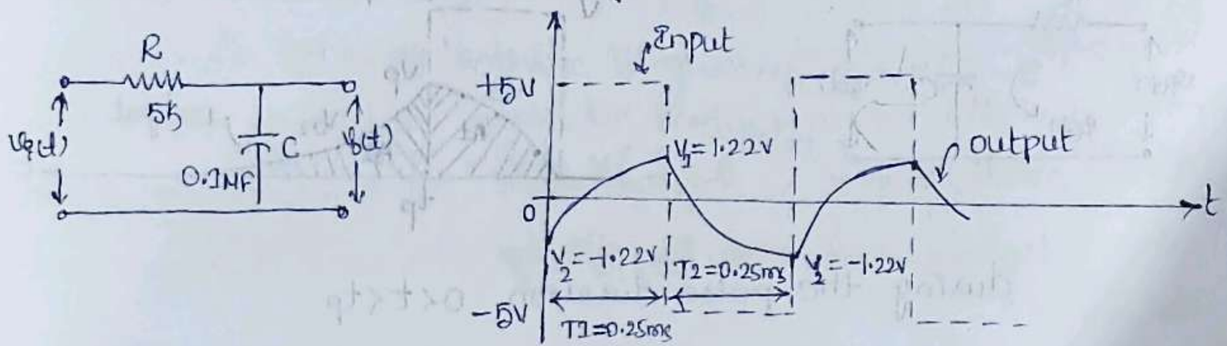
The total area under the waveform $A = A1 + A2$

$$A = Vt_p - V_p RC + V_p RC = Vt_p$$

= The area under the pulse

Since the average voltage across 'R' is zero, the dc voltage at the output is the same as that at the input. So, the area under the o/p waveform is the same as that under the pulse.

Q A symmetrical square wave of amplitude $\pm 5V$ and frequency 2kHz is impressed on an RC low-pass circuit. If $R = 5k\Omega$, $C = 0.1\mu F$ Calculate and plot the steady-state output with respect to time?



when the input waveform is shown by the dotted line in the figure as shown above is applied to the RC lowpass circuit then the capacitor will be charged and discharged exponentially in the output as shown above.

Given $f = 2kHz$, $T = \frac{1}{2kHz} = 0.5ms$

$$T1 = T2 = T/2 = \frac{0.5ms}{2} = 0.25\ ms$$

The time constant of the circuit $\tau = RC = 5 \times 10^{3} \times 0.1 \times 10^{-6}$

$$\tau = 0.5\ ms$$

Since RC is comparable to T/2, (RC = T). the capacitor charges and discharges exponentially. Since the input is symmetrical square wave

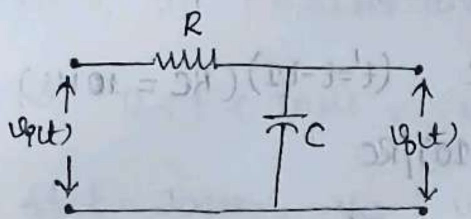
$$V1 = \frac{V}{2}\left(\frac{1-e^{-T/2RC}}{1+e^{-T/2RC}}\right)$$ where 'V' is the peak to peak input

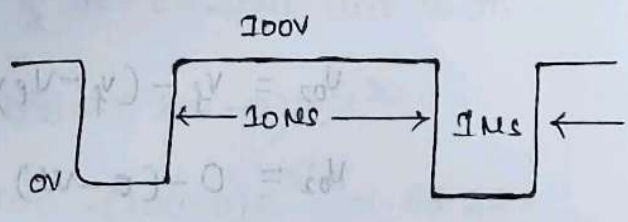
$$V1 = \frac{10}{2}\left(\frac{1-e^{-0.5/1}}{1+e^{-0.5/1}}\right) = \frac{5\times0.393}{1.606} = 1.22V$$

$$V_2 = -V1 = -1.22\,V$$

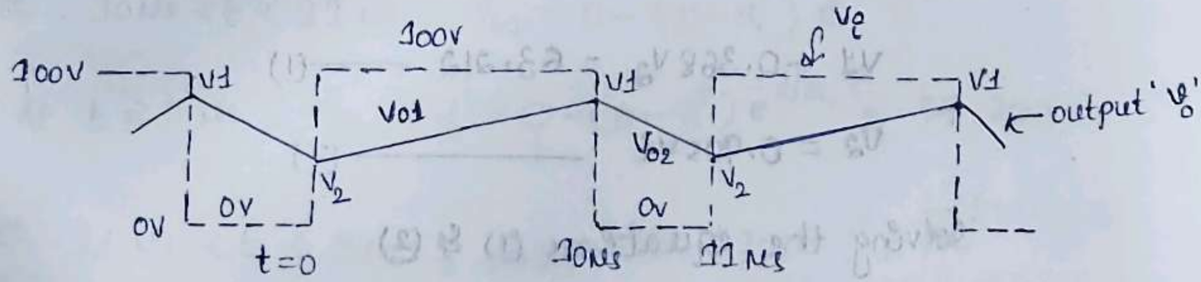
peak to peak value of $o/p = 1.22 - (-1.22) = 2.44\,V$

Q: The periodic waveform shown in figure below is applied to an RC integrating network shown in figure whose time constant is 10μs. Sketch the o/p waveform. Calculate the maximum and minimum values of the o/p voltage with respect to ground under steady-state conditions. Also calculate and plot the o/p for the first two cycles of the input?





Sol:



Steady-state analysis :

Under steady state conditions, the capacitor charges and discharges to the same level in each cycle i.e V_1 and V_2 are constants.

Given $RC = 10\mu s$, $T_{ON} = 10\mu s$, $T_{off} = 1\mu s$

Since $RC \approx T$ i.e time constant is comparable to the period of the waveform, the capacitor charges and discharges gradually and the o/p waveform is as shown below

In the interval $0 < t < 10\mu s$, when input = 100V

$$V_{o1} = V_f - (V_f - V_i)\, e^{-t/RC}$$

$$V_{o1} = 100 - (100 - V_2)\, e^{-t/10} \qquad (RC = 10\mu s)$$

At $t = 10\mu s$, $V_{o1} = V_1 = 100 - (100 - V_2)\, e^{-10/10}$

i.e $V_1 - 0.368 V_2 = 63.212$ —— (1) $\qquad [(1-e^{-1}) = 63.2]$

In the interval $10\mu s < t < 11\mu s$, when input = 0V

$$V_{o2} = V_f - (V_f - V_i)\, e^{-t'/RC} \quad (t' = t - t_1)\ (RC = 10\mu s)$$

$$V_{o2} = 0 - (0 - V_1)\, e^{-(t-10)/RC}$$

At $t = 11\mu s$, $V_{o2} = V_2 = V_1\, e^{-1/10} = 0.905 V_1$ —— (2)

$$V_1 - 0.368 V_2 = 63.212 \quad \text{—— (1)}$$

$$V_2 = 0.905 V_1 \quad \text{—— (2)}$$

Solving the equations (1) & (2)

$$V_1 - 0.368(0.905 V_1) = 63.212$$

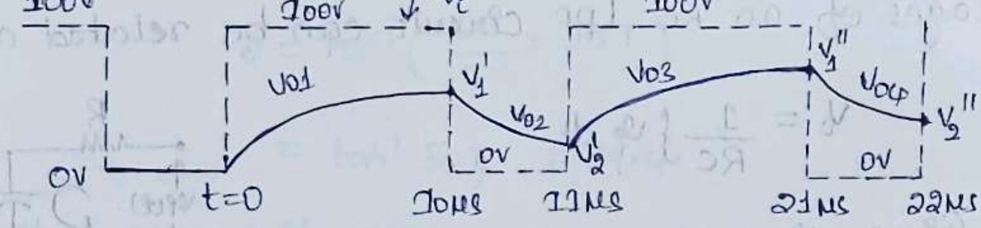
$$V1 = \frac{63.212}{1-0.368\times0.905} = 94.91\ V$$

$$V2 = 0.905\ V1 = 0.905\times94.91 = 85.07\ V$$

The maximum value of o/p voltage $V1 = 94.91\ V$

The minimum value of o/p voltage $V2 = 85.07\ V$

Transient analysis: The o/p waveform for the first two cycles of the input waveform is shown below



Let the capacitor be initially uncharged i.e $V_0 = 0V$ at $t=0$. Since the voltage across the capacitor cannot change instantaneously, the o/p voltage rises from 0V at $t=0$, goes exponentially to v_1' at $t=10\mu s$ to v_2' at $t=11\mu s$ to v_1'' at $t=21\mu s$ and to v_2'' at $t=22\mu s$ and so on ....

For $0 < t < 10\mu s$, $v_{01} = 100-(100-0)e^{-t/10}$

At $t=10\mu s$, $v_{01} = v_1' = 100-(100-0)e^{-10/10} = 63.22\ V$

For $10\mu s < t < 11\mu s$, $v_{02} = 0-(0-v_1')\,e^{-(t-10)/10}$

At $t = 11\mu s$, $v_{02} = v_2' = 0-(0-v_1')\,e^{-1/10} = 57.203\ V$

For $11\mu s < t < 21\mu s$, $v_{03} = 100-(100-v_2')\,e^{-(t-11)/10}$

At $t = 21\mu s$, $v_{03} = v_1'' = 100-(100-v_2')\,e^{-10/10} = 84.255\ V$

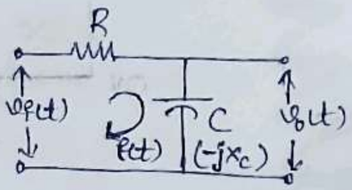
For $21\mu s < t < 22\mu s$, $V_{o4} = 0 - (0 - V_1'')\, e^{-(t-21)/10}$

At $t = 22\mu s$, $V_{o4} = V_2'' = 0 - (0 - V_1'')\, e^{-1/10} = 76.23\,V$

Q8 prove that an RC LPF circuit behaves as a reasonably good integrator if $RC > 15T$, where 'T' is the period of the input sinusoid $E_m \sin\omega t$

Sol: for large time constant $RC > 15T$ the o/p and i/p voltages of an RC LPF circuit can be related as

$$v_o = \frac{1}{RC}\int v_i \, dt$$



Given $v_i = E_m \sin\omega t$

$$v_o = \frac{1}{RC}\int E_m \sin\omega t \, dt$$

$$= -\left(\frac{E_m}{RC\omega}\right)\cos\omega t$$

$$= -\left(\frac{E_m}{RC\omega}\right)\sin(\omega t - \pi/2)$$

from the circuit $v_o = \dfrac{v_i(-jX_c)}{R - jX_c} = \dfrac{v_i/j\omega C}{R + 1/j\omega C}$

$$\frac{v_o}{v_i} = \frac{1}{1 + j\omega RC} \Rightarrow v_o = v_i\left(\frac{1}{1 + j\omega RC}\right)$$

$$v_o = \frac{E_m \sin\omega t}{1 + j\omega RC} = \frac{E_m \sin(\omega t - \theta)}{1 + j\omega RC}$$

where $v_i = E_m \sin\omega t$

$$= \frac{E_m}{\sqrt{1 + (\omega RC)^2}}\sin(\omega t - \theta) \quad \text{where } \theta = \tan^{-1}\omega RC$$

we have $\omega = 2\pi f = \frac{2\pi}{T}$, since $f = 1/T$

$$v_0 = \frac{E_m}{\sqrt{1+(\frac{2\pi}{T})^2 R^2 C^2}} \sin\left(\frac{2\pi}{T}t - \theta\right)$$

comparision of the equations for 'v' reveals that for the RC LPF circuit to act as an integrator, it is essential that the angle 'θ' must be equal to $\pi/2$ radians i.e 90°

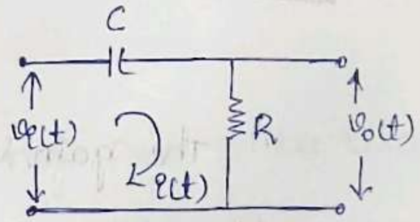
If $RC = 15T$, we have $\theta = \tan^{-1}\omega RC$

$$\theta = \tan^{-1}\left(\frac{2\pi}{T} \times 15T\right)$$

$$= \tan^{-1} 30\pi = 89.4^\circ$$

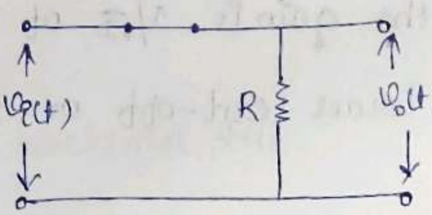
That means if $RC > 15T$, angle 'θ' becomes almost 90°. So this is the condition to be satisfied for the RC low pass circuit to behave like a reasonably good integrator

The High-pass RC circuit :



The high pass 'RC' circuit is as shown in the figure. At Lower frequencies the reactance of the capacitor is infinity and it blocks the input and hence the o/p is zero.

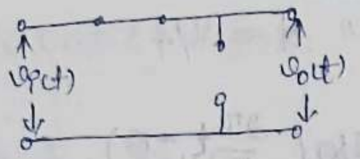
(i) $X_C = 1/2\pi fc$



At $f=\infty$; $X_C = \frac{1}{\infty} = 0$

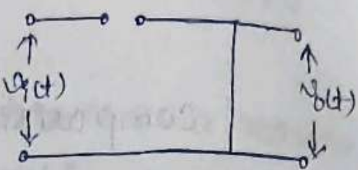
and if $R=\infty$ Resistor is opened

i.e



$\therefore\ v_o(t) = v_i(t)$ for $f = \infty$

(ii) when $f = 0$; $X_C = \frac{1}{2\pi f C} = \infty$

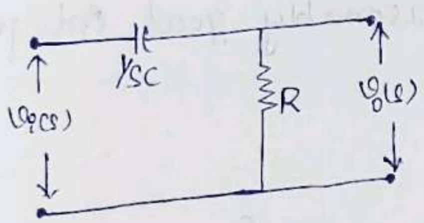


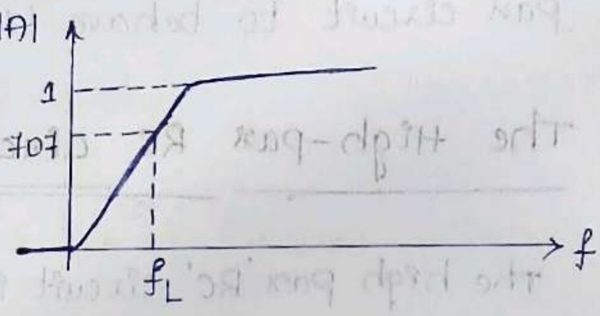
and If $R = 0\,\Omega$; Resistor is short circuited

$v_o(t) = 0V$ for $f = 0$

so from the above two cases when $f = \infty$; $v_o(t) = v_i(t)$ i.e o/p is following the input. when $f = 0$; $v_o(t) = 0V$ i.e output is low. so from these two conditions the circuit is allowing only high frequency's and attenuating low frequencies.

Sinusoidal Input :





The gain versus frequency curve of a high pass circuit excited by a sinusoidal input is shown in figure. for a sinusoidal input v_i, the output signal v_o increases in amplitude with increasing frequency. The frequency at which the gain is $1/\sqrt{2}$ of its maximum value is called the lower cut-off or 3-db lower frequency.

From the figure $v_o(s) = \dfrac{v_i(s)\, R}{R + 1/sC}$

$$\frac{v_o(s)}{v_i(s)} = \frac{RSC}{SCR+1} = \frac{j\omega RC}{1+j\omega RC}$$

$$A = \frac{1}{1+(1/j\omega RC)} = \frac{1}{1-j(1/\omega RC)}$$

$$A = \frac{1}{1-j\left(\frac{1}{2\pi f RC}\right)} = \frac{1}{1-j\left(\frac{f_L}{f}\right)}$$

$$|A| = \frac{1}{\sqrt{1+(f_L/f)^2}} \qquad \text{where } f_L = 1/2\pi RC$$

'f_L' is called as the lower cut-off frequency.

At $f = f_L$, $|A| = \frac{1}{\sqrt{2}}$ Lower 3-db frequency.

Relation between f_L and tilt:

The lower cut-off frequency of high pass filter circuit is $f_L = 1/2\pi RC$. The lower cut-off frequency produces a tilt. For a 10% change in capacitor voltage, the time or pulse width involved is

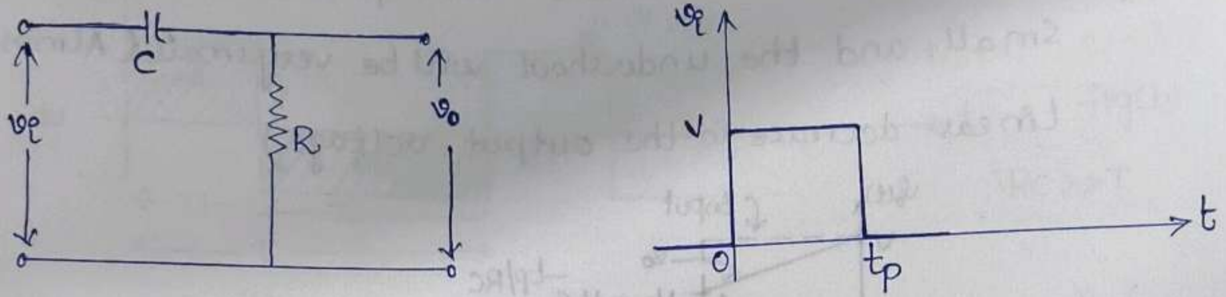
$$t = 0.1RC = PW$$

$$\frac{\text{pulse width}}{RC} = 0.1 = \text{fractional tilt}$$

$$\text{fractional tilt} = \frac{PW}{RC} = \frac{2\pi(PW)}{2\pi RC} = 2\pi f_L\,(PW)$$

for $t > 5\tau$, the output will reach more than 99% of its final value. Hence although the steady state is approached asymptotically, for most applications we may assume that the final value has been reached after 5τ.

pulse Input :

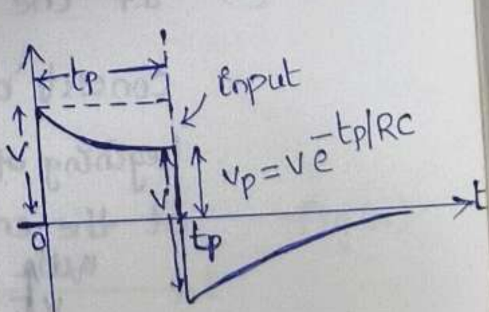


An ideal pulse has the waveform as shown above. The pulse amplitude is 'V' and the pulse duration is 't_p'.

Case (i): The respone for $0<t<t_p$ is same as the step input. Hence the output is given by $v_o = V e^{-(t/RC)}$. The output at $t=t_p$ is given by $v_o = V e^{-(t_p/RC)} \approx V_p$. At the end of the pulse, the input falls abruptly the amount V, but the voltage across the capacitor cannot change instantaneously, the o/p falls suddenly by 'V' to $V_p - V$ volts.

Case (ii): At $t = t_p^+$, $v_o = V_p - V$.

Since $V_p < V$ the voltage becomes negative and then decays exponentially to zero as indicated in figure.

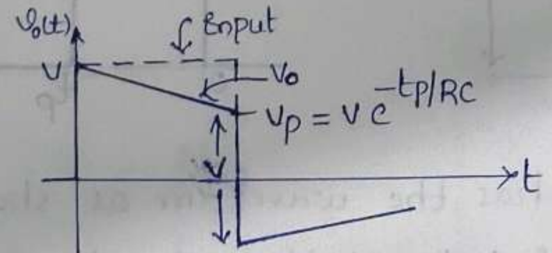


The output voltage is given by

$$v_o = (Ve^{-t_p/RC} - V)\, e^{-(t-t_p)/RC}$$

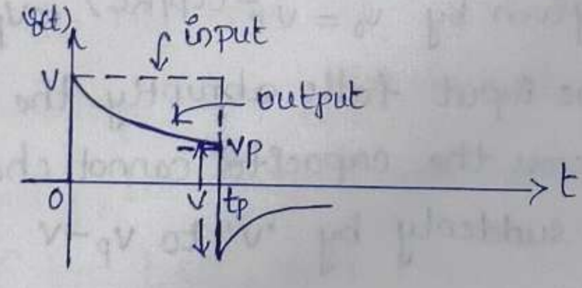
The response for output waveforms $RC >> t_p$, $RC \approx t_p$ and $RC << t_p$ are shown in below figures. There is positive area and negative area in the output waveforms. The negative area will always be equal to the positive area.

(i) If the time constant $RC >> t_p$ the tilt will be very small and the undershoot will be very small (Almost Linear decrease in the output voltage)



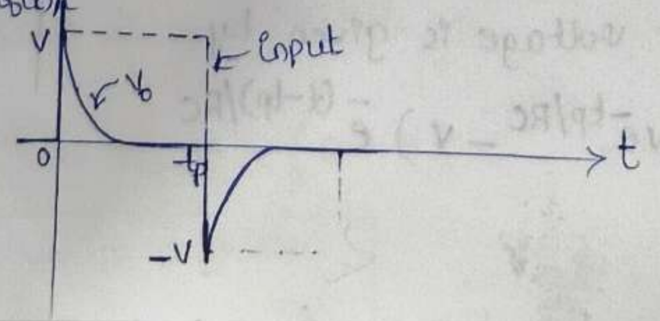
$\frac{RC}{t_p} >> 1$

(ii) If the time constant is $RC \approx t_p$ the output rises towards zero level very very slowly.



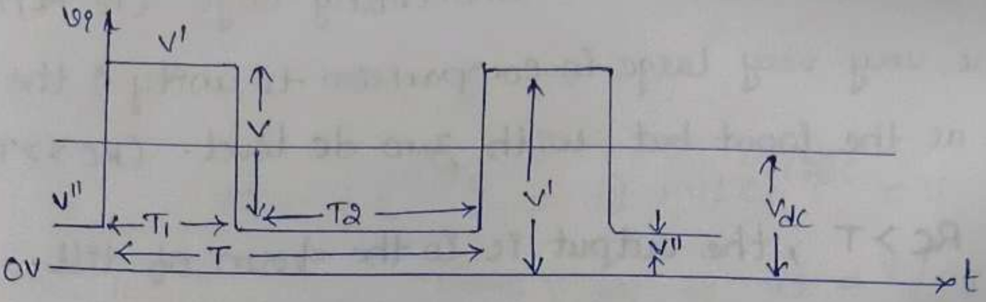
$\frac{RC}{t_p} \approx 1$

(iii) If the time constant is $RC << t_p$ the output consists of a positive spike of amplitude 'V' volts at the begining of the pulse and a negative spike ~~of~~ same amplitude at the end of the pulse. It is called "peaking".

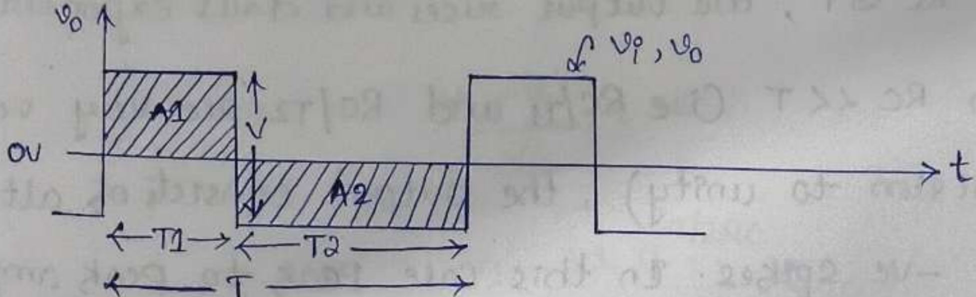


$\frac{RC}{t_p} << 1$

Square wave :

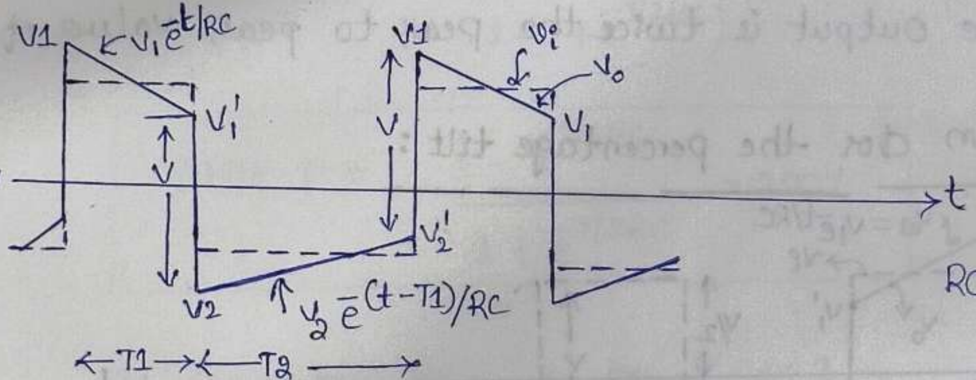


fig(a)



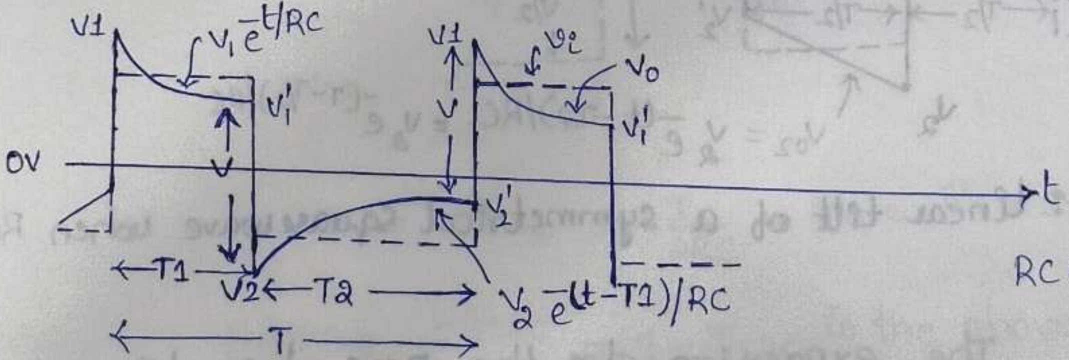
fig(b)

RC >> T



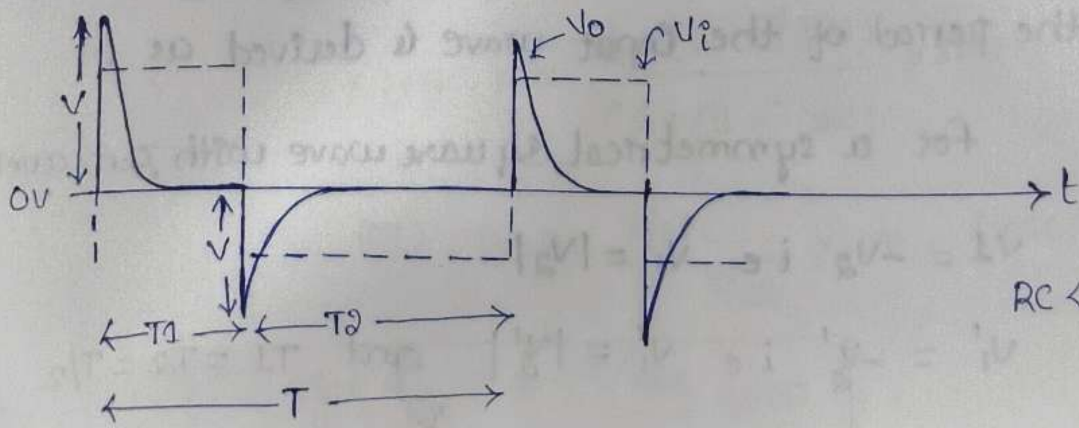
fig(c)

RC > T



fig(d)

RC ≈ T



fig(e)

RC << T

(i) when the time constant is arbitrarily large (i.e RC/T_1 and RC/T_2 are very very large in comparision to unity) the output is same as the input but with zero dc level. $(RC>>T)$

(ii) When $RC > T$, the output is in the form of tilt.

(iii) When $RC \approx T$, the output rises and falls exponentially.

(iv) When $RC << T$ (i.e RC/T_1 and RC/T_2 are very very small in comparision to unity), the output consists of alternate +ve and −ve spikes. In this case peak to peak amplitude of the output is twice the peak to peak value of the input.

Expression for the percentage tilt :

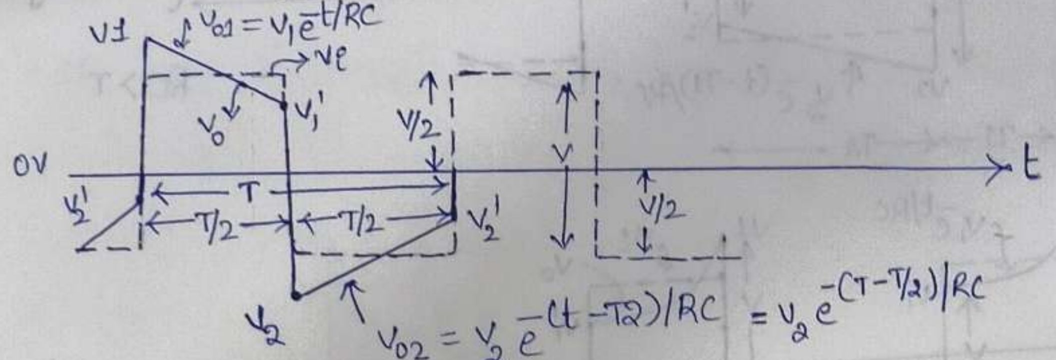


fig: Linear tilt of a symmetrical squarewave when $RC>>T$

The expression for the percentage tilt when the time constant RC of the circuit is very large compared to the period of the input wave is derived as,

for a symmetrical square wave with zero average value

$V_1 = -V_2$ i.e $V_1 = |V_2|$

$V_1' = -V_2'$ i.e $V_1' = |V_2'|$ and $T_1 = T_2 = T/2$

$$v_1' = v_1 e^{-T/2RC} \quad \text{and} \quad v_2' = v_2 e^{-T/2RC}$$

$$v_1 - v_2' = V$$

$$v_1 - v_2 e^{-T/2RC} = v_1 + v_1 e^{-T/2RC} = V$$

$$v_1 (1 + e^{-T/2RC}) = V \Rightarrow v_1 = V/(1 + e^{-T/2RC})$$

% tilt, $P = \dfrac{v_1 - v_1'}{V/2} \times 100\%$

$$\boxed{= \frac{v_1 - v_1 e^{-T/2RC}}{v_1 (1 + e^{-T/2RC})} \times 200\%}$$

$$\boxed{\%\ \text{tilt}\ P = \frac{1 - e^{-T/2RC}}{1 + e^{-T/2RC}} \times 200\%}$$

If $RC >> T$, $\dfrac{RC}{T} >> 1$ or $\dfrac{T}{RC} << 1$

Also $\dfrac{T}{2RC} << 1$

$\therefore e^{-T/2RC} \cong 1 - \dfrac{T}{2RC}$, since $e^{-x} = 1 + (-x)$ for $x << 1$

putting $e^{-T/2RC} = 1 - T/2RC$ in the above expression for 'P', we get

$$\%\ \text{tilt}\ P = \frac{1 - (1 - T/2RC)}{1 + (1 - T/2RC)} \times 200$$

(or) $P = \dfrac{T/2RC}{2 - T/2RC} \times 200$, since $T/2RC << 1$ can be ignored

$$\boxed{\%\ \text{Tilt},\ P = \frac{T/2RC}{2} \times 200}$$

$$v_1' = v_1 e^{-T/2RC} \quad \text{and} \quad v_2' = v_2 e^{-T/2RC}$$

$$v1 - v_2' = v$$

$$v1 - v_2 e^{-T/2RC} = v1 + v1 e^{-T/2RC} = v$$

$$v_1 (1 + e^{-T/2RC}) = v \Rightarrow v1 = v/(1 + e^{-T/2RC})$$

$$\%\text{ tilt},\ P = \frac{v_1 - v_1'}{v/2} \times 100\%$$

$$\boxed{= \frac{v_1 - v1\, e^{-T/2RC}}{v1(1 + e^{-T/2RC})} \times 200\%}$$

$$\boxed{\%\text{ tilt } P = \frac{1 - e^{-T/2RC}}{1 + e^{-T/2RC}} \times 200\%}$$

If $RC >> T$, $\frac{RC}{T} >> 1$ or $\frac{T}{RC} << 1$

Also $\frac{T}{2RC} << 1$

$\therefore e^{-T/2RC} \cong 1 - \frac{T}{2RC}$, since $e^{-x} = 1 + (-x)$ for $x << 1$

putting $e^{-T/2RC} = 1 - T/2RC$ in the above expression for 'P', we get

$$\%\text{ tilt } P = \frac{1 - (1 - T/2RC)}{1 + (1 - T/2RC)} \times 200$$

(or) $P = \frac{T/2RC}{2 - T/2RC} \times 200$, since $T/2RC << 1$ can be ignored

$$\boxed{\%\text{ Tilt},\ P = \frac{T/2RC}{2} \times 200}$$

$$\% \text{ tilt} = \frac{T}{\frac{4RC}{2}} \times 100\,\%$$

$$= \frac{T}{2RC} \times 100\,\%$$

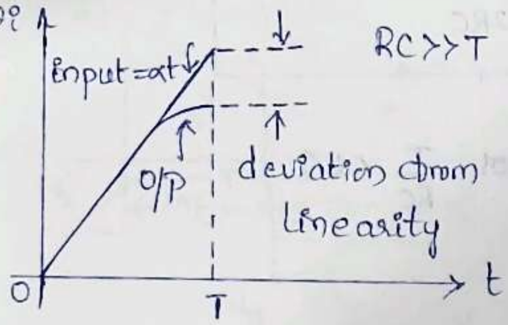
$$= \frac{T \times \pi}{2\pi RC} \times 100\,\%$$

$$= \frac{\pi}{f}\left(\frac{1}{2\pi RC}\right) \times 100\,\%$$

$$\boxed{\% \text{ tilt} = \frac{\pi}{f} \cdot f_L \times 100\,\%}$$

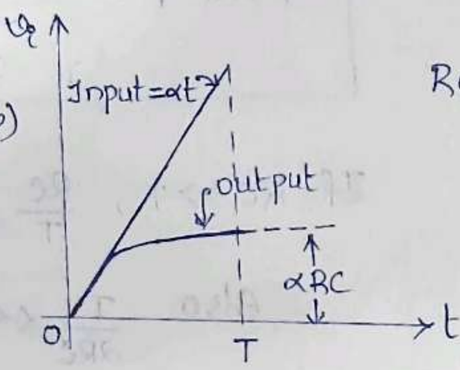
Ramp Input :

(a)



$RC >> T$

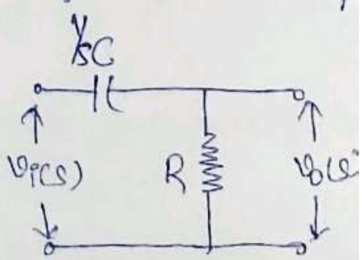
(b)



$RC << T$

A waveform which is zero for $t<0$ and which increases linearly with time for $t>0$ is called a ramp or sweep voltage. when the highpass RC circuit is excited by a ramp input $v_i(t) = \alpha t$, where 'α' is the slope of the ramp then

$$v_i(s) = \alpha / s^2$$



From laplace transformed circuit

$$v_o(s) = \frac{v_i(s)\, R}{R + 1/sC}$$

$$v_o(s) = \frac{\alpha}{s^2} \cdot \frac{RCs}{1+RCs} = \frac{\alpha RC}{sRC\,[s+1/RC]}$$

$$v_o(s) = \frac{\alpha}{s(s+1/RC)} = \alpha RC\left[\frac{1}{s} - \frac{1}{s+1/RC}\right]$$

$$v_o(s) = \alpha RC\left[1/s - 1/s+\tfrac{1}{RC}\right]$$

Taking the inverse Laplace transform on both sides

$$v_o(t) = \alpha RC\left[1 - e^{-(\frac{1}{RC})t}\right]$$

$$= \alpha RC\left[1 - \left(1 + \left(\frac{-t}{RC}\right) + \left(\frac{-t}{RC}\right)^2\frac{1}{2!} + \cdots\right)\right]$$

$$= \alpha RC\left(+\frac{t}{RC}\right) - \frac{\alpha RC}{2}\left(+\frac{t}{RC}\right)^2$$

$$= \alpha t - \frac{\alpha t^2}{2RC} = \alpha t\left[1 - \frac{t}{2RC}\right]$$

$$\boxed{v_o(t) = \alpha t\,(1 - t/2RC)}$$

The response of highpass circuit for a ramp input when $RC >> T$ and $RC << T$ are shown in figures a&b. The output signal falls away slightly from the input when 'T' is small.

When a ramp signal is transmitted through a linear network, the o/p departs from the input. A measure of the departure from linearity expressed as the transmission error 'e_t'. It is defined as the difference b/w the input and the o/p divided by input.

$$e_t = \left.\frac{v_i - v_o}{v_i}\right|_{t=T}$$

$$e_t = \left.\frac{\alpha t - \alpha t\left(1 - \frac{t}{2RC}\right)}{\alpha t}\right|_{t=T} = \left.\frac{\alpha t^2}{2RC(\alpha t)}\right|_{t=T}$$

$$e_t = \frac{T}{2RC} = \pi f_L T.$$

The High pass RC circuit as a differentiator:

When the time constant of the high pass RC circuit is very very small, the capacitor charges very quickly. So almost all the input $v_i(t)$ appears across the capacitor and the voltage across the resistor will be negligible compared to the voltage across the capacitor. Then the current is determined entirely by the capacitance. Then the current

$$i(t) = C\frac{dv_i(t)}{dt}$$

The output signal across 'R' is

$$v_o(t) = i(t)R = RC\frac{dv_i(t)}{dt}$$

Thus the output is proportional to the derivate of Input.

"The high pass RC circuit acts as a differentiator provided the RC time constant of the circuit is very small in comparision with the time required for the input signal to make an appreciable change".

... coupling capacitor $C_c = 1\mu F$. when the input resistance $R_i = 1M\Omega$, determine the minimum square wave frequency that can be displayed if the tilt is not to exceed 1%. ?

Sol:

$$\text{fractional tilt} \times R_i C_c = PW$$

$$PW = 0.01 \times 1\times10^{6} \times 1\times10^{-6} = 0.01 \text{ sec}$$

$$f = \frac{1}{T} = \frac{1}{2PW} = \frac{1}{2\times0.01} = 50\text{Hz}$$

Q A 10Hz square wave is fed to an amplifier. Calculate and plot the output waveform under the following con... The lower 3db frequency is (a) 0.3 Hz (b) 3 Hz and (c) 3...

Sol: Since the lower cut-off frequency is specified, the amplifier acts as an RC high-pass circuit.

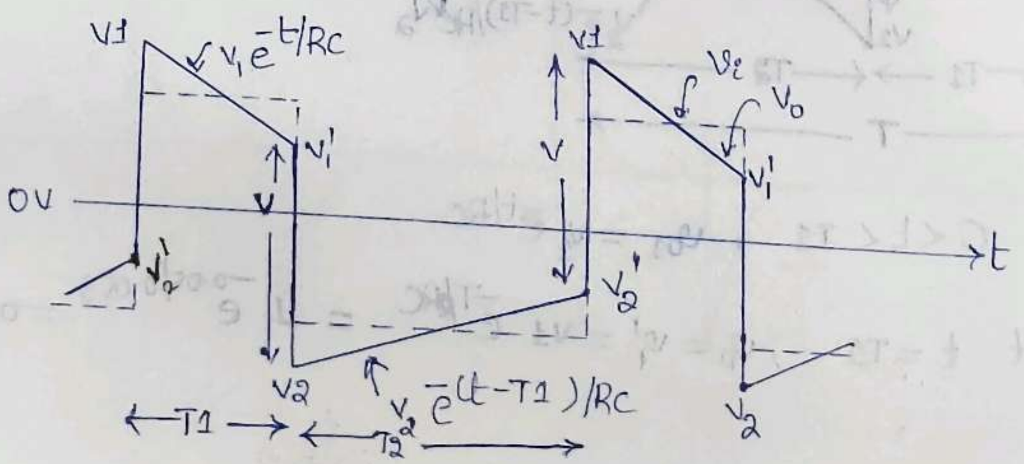
Given input frequency $f_i = 10\text{Hz}$

$$T = 1/f = 1/10 = 0.1 \text{ sec}$$

(a) when $f_L = 0.3\text{Hz}$; $T = 0.1$ sec

Time constant $RC = \frac{1}{2\pi f_L} = \frac{1}{2\pi\times0.3} = 0.5304$ se...

$RC > T$



Q An oscilloscope has a coupling capacitor $C_c = 1\mu F$. when the input resistance $R_i = 1M\Omega$, determine the minimum square wave frequency that canbe displayed if the tilt is not to exceed 1%. ?

Sol:

$$\text{fractional tilt} \times R_i C_c = PW$$

$$PW = 0.01 \times 1\times10^{6} \times 1\times10^{-6} = 0.01 \text{ sec}$$

$$f = \frac{1}{T} = \frac{1}{2PW} = \frac{1}{2\times0.01} = 50\,Hz$$

Q A 10Hz square wave is fed to an amplifier. Calculat and plot the output waveform under the following condi The lower 3db frequency is (a) 0.3 Hz (b) 3 Hz and (c) 30.

Sol: Since the lower cut-off frequency is specified, the amplifier acts as an RC high-pass circuit.

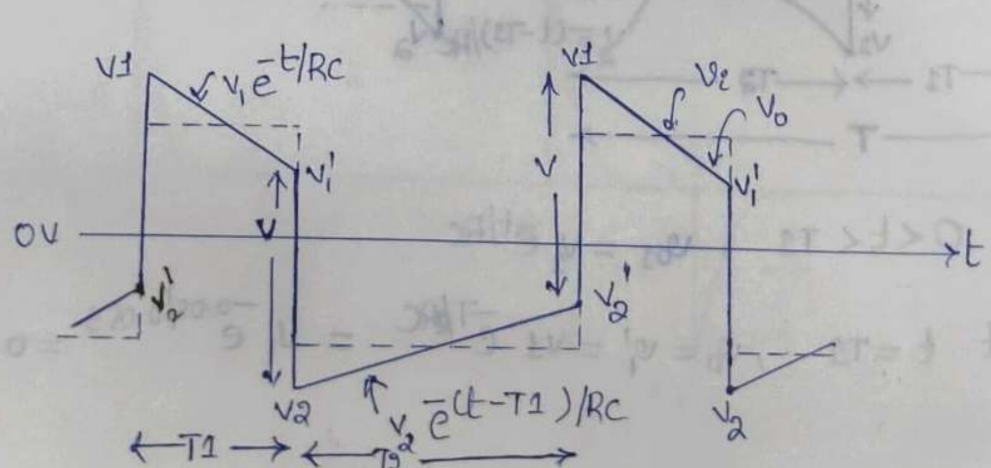
Given input frequency $f_i = 10\,Hz$

$$T = 1/f = 1/10 = 0.1 \text{ sec}$$

(a) when $f_L = 0.3\,Hz$; $T = 0.1$ sec

$$\text{Time constant } RC = \frac{1}{2\pi f_L} = \frac{1}{2\pi\times0.3} = 0.5304 \text{ sec}$$

$$RC > T$$



$V1 = \dfrac{V}{1+e^{-T/2RC}}$, where 'V' is the peak to peak value of the input voltage. Substituting values of T and RC,

$$V1 = \frac{V}{1+e^{-0.1/2\times0.5304}} = 0.5234\ V$$

$$V_1' = V1\, e^{-T1/RC} = V1\, e^{-T/2RC} = (0.5234V)e^{-0.1/2\times0.5304}$$

$$V_1' = 0.4765\ V \text{ volts.}$$

Also $V_2 = -V1 = -0.5234\ V$ volts

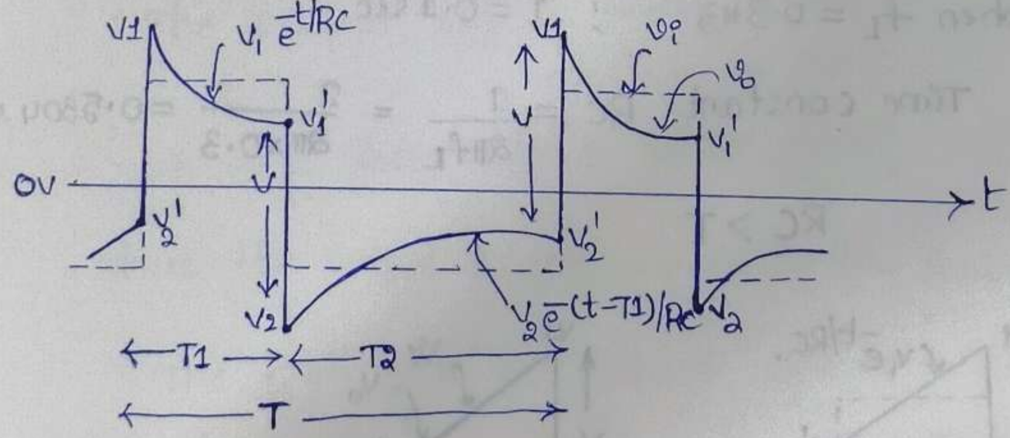
$$V_2' = -V_1' = -0.4765\ V \text{ volts}$$

(b) when $f_L = 3Hz$: $T = 0.1$ sec

$$RC = \frac{1}{2\pi f_L} = \frac{1}{2\pi\times3} = 0.053 \text{ sec}$$

$$T/2 = 0.05 \text{ sec}$$

Since $RC \approx T$, the output rises and decays exponentially



for $0 < t < T1$, $v_{o1} = V_1 e^{-t/RC}$

At $t = T1$, $v_o = v_1' = V1\, e^{-T1/RC} = V_1\, e^{-0.05/0.053} = 0.389 V1$

For $T1 < t < T1+T2$, $v_{o2} = V_2 e^{-(t-T1)/RC}$

At $t=T$, $v_o = v_2' = V_2 e^{-T2/RC} = V_2 e^{-0.05/0.053} = 0.389 V_2$

Since peak to peak input is 'V' volts

$V_1' - V_2 = V$ i.e $0.389 V1 - V_2 = V$

$V1 - V_2' = V$ i.e $V1 - 0.389 V_2 = V$

$0.389 V1 - V_2 = V1 - 0.389 V_2$

$V1 = -V_2$

$V1 = \frac{V}{1.389} = 0.7199 V$ volts

$V_2 = -V1 = -0.7199V$ volts

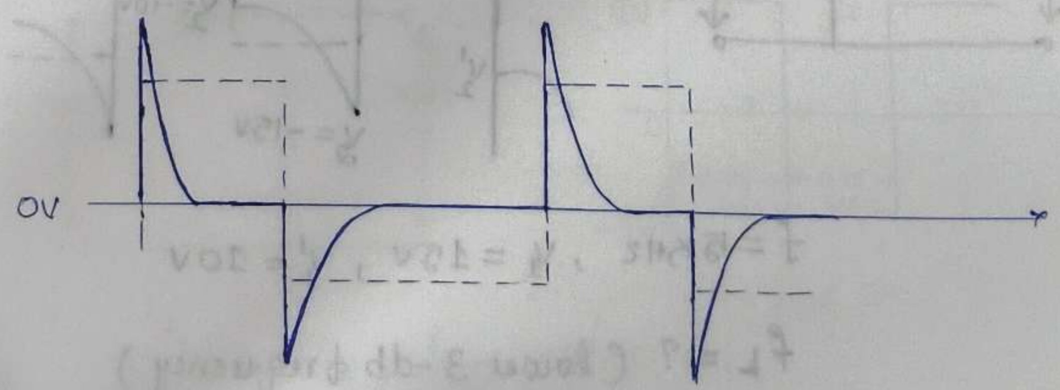
$V_1' = 0.389 V1 = 0.389 \times 0.7199V = 0.280 V$

$V_2' = -V_1' = -0.280 V$

(C) when $f_L = 30Hz$; $T = 0.1$ sec

Time constant $RC = \frac{1}{2\pi f_L} = \frac{1}{2\pi \times 30} = 0.0053$ sec

since $RC << T$ the output is in the form of sp



Q: Calculate the lowest square wave frequency that can be passed by an amplifier with a lower cut-off frequency of 10Hz, if the o/p tilt is not to exceed 2% ?

Sol: Since the lower cut-off frequency is given, the circuit is high-pass filter.

Given $f_L = 10\,Hz$ and o/p tilt = 2% (or) 0.02

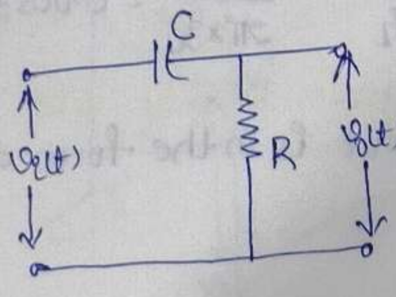
$$\%\ tilt = \left(\frac{\pi f_L}{f}\right) \times 100$$

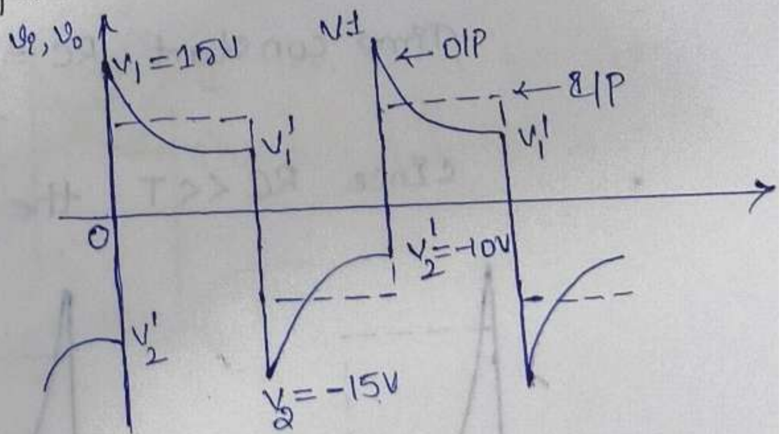
$$2 = \left(\frac{\pi \times 10}{f}\right) \times 100$$

$$f = \frac{\pi \times 10 \times 100}{2} = 1570.8\,Hz$$

Q: If a square wave of 5KHz is applied to an RC high-pass circuit and the resultant waveform measured on a CRO was tilted from 15V to 10V, find out the lower 3-db frequency of the high pass circuit?

Sol:





$f = 5\,KHz$, $V_1 = 15V$, $V_1' = 10V$

$f_L = ?$ (lower 3-db frequency)

Peak to peak value of Input $V = V_1 + V_2 = 15 + 15 = 30\,V$

$$\% \text{ tilt } P = \frac{V_1 - V_1'}{V/2} \times 100$$

$$= \frac{15 - 10}{\left(\frac{30}{2}\right)} \times 100$$

$$= 33.33\,\%$$

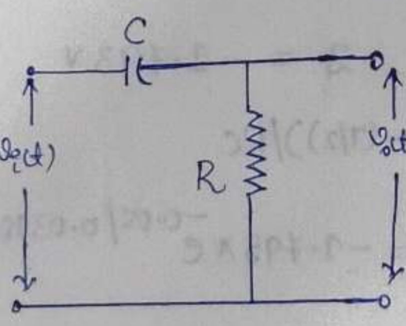
Also $\% \text{ tilt } P = \frac{\pi f_L}{f} \times 100$

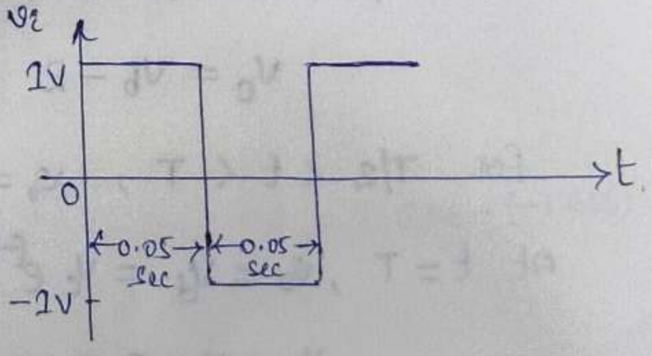
$$33.33 = \frac{3.141 \times f_L}{5 \times 10^3} \times 100$$

$$f_L = \frac{33.33 \times 5 \times 10^3}{3.141 \times 100} = 530.56\,Hz$$

Q A 10 Hz symmetrical square wave whose peak-to-peak amplitude is 2V is impressed upon a high-pass RC circuit whose 3-db frequency is 5 Hz. Calculate and sketch the o/p waveform for the first two cycles. What is the peak to peak output amplitude under steady state conditions?

Sol:





$$f_L = \frac{1}{2\pi RC} = 5\,KHz$$

$$RC = \frac{1}{2\pi f_L} = \frac{1}{2\pi \times 5K} = 0.0318\,sec$$

Input signal frequency $f = 10\,Hz$

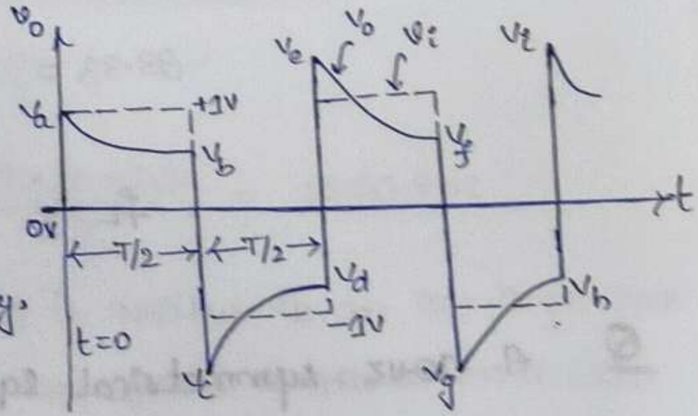
Time period $T = 1/f = \frac{1}{10} = 0.1$ sec

$$T/2 = \frac{0.1}{2} = 0.05\,\text{sec}$$

$RC \approx T/2$ i.e RC is comparable to $T/2$, so the capacitor charges appreciably in half cycle.

Transient response :

The transient response is shown in figure. Assume that the capacitor is uncharged initially. So $v_o = 0$ for $t<0$. Since at $t=0$, the input rises to $+1V$ abruptly, the o/p also rises abruptly by $+1V$.



At $t=0$, $V_o = V_a = +1V$

for $0<t<T/2$, $v_o = V_a e^{-t/RC}$

At $t = T/2$, $v_o = V_b = V_a e^{(-T/2)RC} = 1 \times e^{-0.05/0.0318} = 0.207\,V$

$$V_c = V_b - 2 = 0.207 - 2 = -1.793\,V$$

for $T/2 < t < T$, $v_o = V_c e^{-(t-(T/2))/RC}$

At $t = T$, $v_o = V_d = V_c e^{(-T/2)/RC} = -1.793 \times e^{-0.05/0.0318} = -0.372V$

$$V_e = V_d + 2 = -0.372 + 2 = 1.628\,V$$

For $T < t < 3T/2$, $v_o = V_e e^{-(t-T)/RC}$

At $t = 3T/2$, $v_o = V_f = V_e e^{(-T/2)/RC} = 1.628 \times e^{-0.05/0.318} = 0.338\,V$

$v_g = v_f - 2 = 0.338 - 2 = -1.662\ V$

For $3T/2 < t < 2T$, $v_o = v_g\, e^{-(t-(3T/2))/RC}$

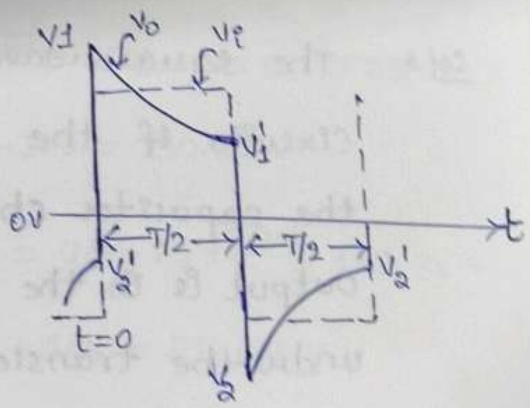
At $t = 2T$, $v_o = v_h = -1.662 \times e^{(-T/2)/RC} = -1.662 \times e^{-0.005/0.0318}$

$v_o = -0.345\ V$

$v_i = v_h + 2 = -0.345 + 2 = 1.655\ V$

Steady-state analysis :

Under the steady-state conditions the capacitor charges and discharges to the same levels in each cycle. Since the input is symmetrical square wave,



$V1 = |V_2|$ i.e $V1 = -V_2$

and $V_1' = |V_2'|$ i.e $V_1' = -V_2'$

The peak to peak value of input $= V$

hence $V1 = \dfrac{V}{1 + e^{(-T/2)/RC}} = \dfrac{2}{1 + e^{-0.05/0.0318}} = 1.656\ V$

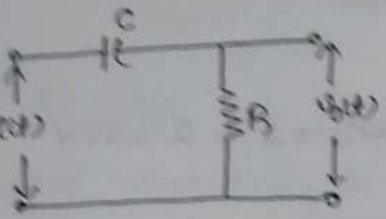
$V2 = -V1 = -1.656\ V$

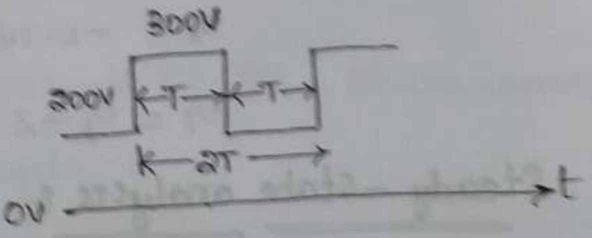
peak to peak value of o/p $= V1 - V2 = 1.656 - (-1.656)$
$= 3.312\ V$

$V_1' = V_1\, e^{-(T/2)/RC} = 1.656\, e^{-0.05/0.0318} = 0.344\ V$

$V_2' = -V_1' = -0.344\ V$

5. The square wave shown in figure is fed to an RC coupling network. what are the output voltage waveforms if (a) RC is very large RC = 10T and (b) RC is very small RC=T/10

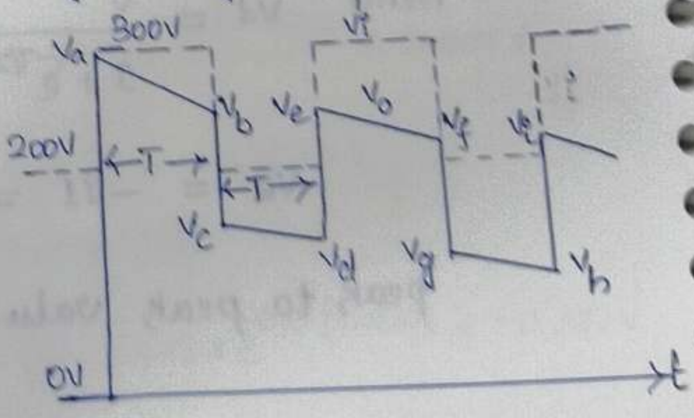




Sol: The square wave is applied to the RC high-pass circuit, if the time constant is very large (RC=10T) the capacitor charges and discharges very slowly. The output is in the form of a tilt. The output waveform under the transient and steady-state conditions are shown below.

(a) Transient response:

When time constant is very large (RC = 10T) the capacitor charges and discharges very slowly. The output is in the form of a tilt:



Since the voltage across the capacitor cannot change instantaneously, v_o is also equal to 300V at t=0

At $t=0$, $v_o = V_a = 300V$

for $0<t<T$, $v_o = V_a e^{-t/RC}$

At $t=T$, $v_0 = V_b = V_a e^{-T/RC} = 300 \times e^{-T/10T} = 271.45\text{ V}$

$V_c = V_b - 100 = 271.45 - 100 = 171.45\text{ V}$

for $T < t < 2T$, $v_0 = V_c e^{-(t-T)/RC}$

At $t = 2T$, $v_0 = v_d = V_c e^{-T/10T} = 171.45 \times e^{-0.1} = 155.129\text{ V}$

$V_e = V_d + 100 = 155.129 + 100 = 255.129\text{ V}$

for $2T < t < 3T$, $v_0 = V_e e^{-(t-2T)/RC}$

At $t = 3T$, $v_0 = V_f = V_e e^{-T/10T} = 255.129 \times e^{-0.1} = 230.84\text{ V}$

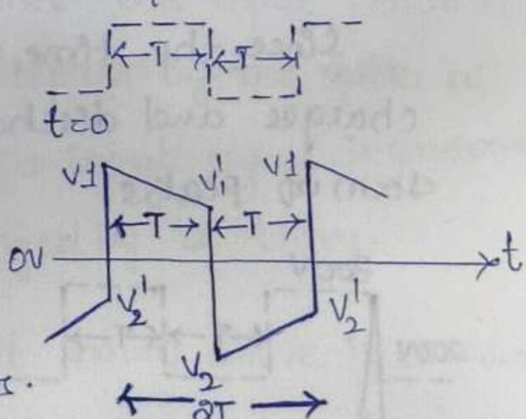
$V_g = V_f - 100 = 230.84 - 100 = 130.84\text{ V}$

for $3T < t < 4T$, $v_0 = V_g e^{-T/10T} = 130.84 \times e^{-0.1} = 118.38\text{ V}$

$V_e = V_h + 100 = 118.38 + 100 = 218.38\text{ V}$

Steady-state response:

Under steady-state the output waveform is symmetrical with respect to zero voltage line because the capacitor blocks dc and the average value of the output is '0' volts.



for $0 < t < T$, $v_0 = V_1 e^{-t/RC}$

At $t = T$, $v_0 = V_1' = V_1 e^{-T/10T} = 0.905\, V_1$

for $T < t < 2T$, $v_0 = V_2 e^{-(t-T)/RC}$

At $t = 2T$, $v_0 = V_2' = V_2 e^{-T/10T} = 0.905\, V_2$

for a symmetrical square wave Input

$$v_1' = -v_2' \text{ and } v_1 = -v_2$$

$$v_1' - v_2 = 100 \text{ and } v_1 - v_2' = 100$$

i.e $0.905\, v_1 - v_2 = 100$ and $v_1 - 0.905\, v_2 = 100$

solving the above two equations;

$$0.905(100 + 0.905\, v_2) - v_2 = 100 \text{ i.e } -0.18\, v_2 = 9.5$$

$$v_2 = 9.5/(-0.18) = -52.49 \text{ V}$$

$$v_1 = 0.905 \times (-52.49) + 100 = 52.49 \text{ V}$$

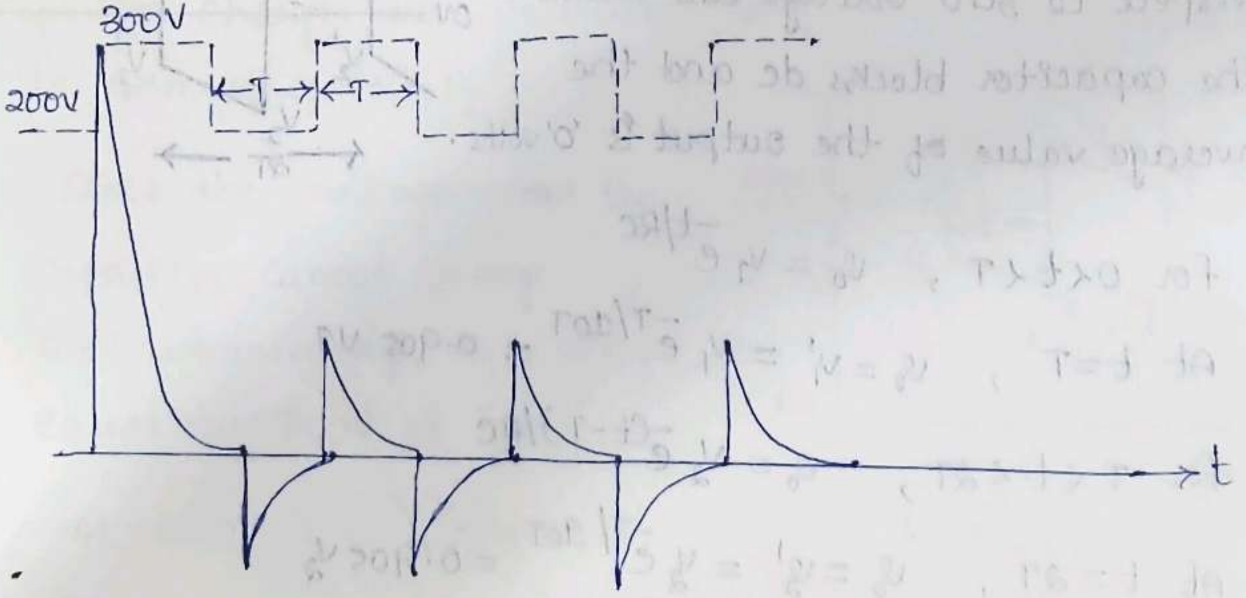
i.e $v_1 = 52.49$ V , $v_2 = -52.49$ V

$$v_1' = 0.905 \times 52.49 = 47.59 \text{ V}$$

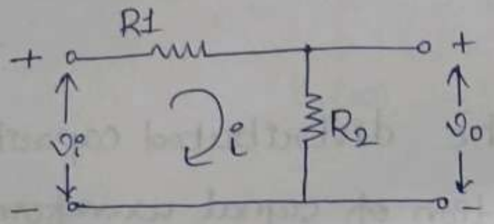
$$v_2' = 0.905 \times (-52.49) = -47.59 \text{ V}$$

(b) Time constant is very small, $RC = T/10$

Since the time constant is very small, the capacitor charges and discharges very fast. The output is in the form of peaks.



Attenuators : Attenuators are resistive networks, which are used to reduce the amplitude of the input signal. The attenuator is basically a potential divider arrangement as shown below figure



v_i = input voltage and
v_o = output voltage
i = circuit current $= \frac{v_i}{R_1+R_2}$

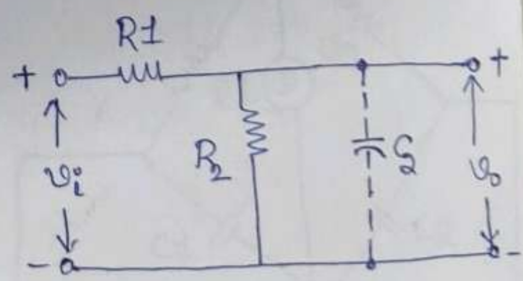
output voltage $v_o = iR_2 = \frac{v_i}{R_1+R_2} \cdot R_2$

$$v_o = \left(\frac{R_2}{R_1+R_2}\right) v_i = a v_i$$

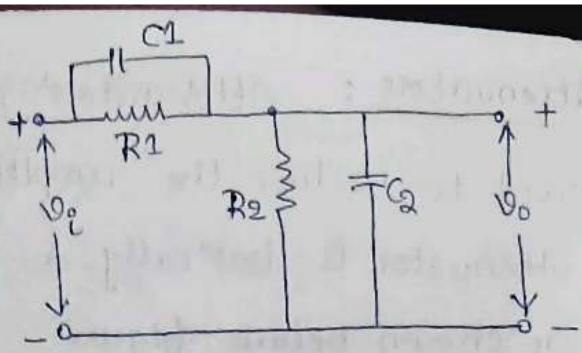
where $a = \frac{R_2}{R_1+R_2}$

hence output is 'a' times the input. Another words, the input signal gets multiplied by the ratio 'a'. Theoretically, the waveform of the input signal is preserved although its amplitude has changed by 'a' times.

The attenuator circuit shown above is considered to be a purely resistive circuit. But in practice there exists a stray capacitance shunting the resistor 'R_2' of the attenuator. The effect of the stray capacitance is that, it distorts the waveshape of the input signal. This effect of 'C_2' is normally overcome by providing necessary and adequate compensation.



So, in a compensated attenuator, a capacitor C1 is put across the resistor R1 and its value is such that $R1C1 = R2C2$

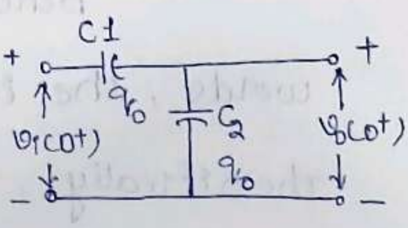


The undesirable effect of the distributed capacitance C2 can be neutralised and distortion of signal waveform prevented, if $R_1C_1 = R_2C_2$

Proof:

(i) At $t = (0^+)$ i.e immediately after the input signal is applied, the capacitors acts as short circuits and hence the resistors go out of circuit.

The capacitors get charged. Let 'q_o' denotes the charge on each capacitor at $t = t(0^+)$. We have $q = CV$,



In general $q_o = C_2 V_o(0^+)$ (or) $V_o(0^+) = q_o / C_2$

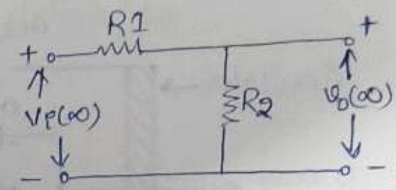
Also $v_i(0^+) = \dfrac{q_o}{C1} + \dfrac{q_o}{C_2} = q_o\left[\dfrac{1}{C1} + \dfrac{1}{C_2}\right]$

$$v_i(0^+) = q_o\left[\frac{C_1 + C_2}{C1\,C_2}\right]$$

$$\text{initial attenuation} = \frac{v_o(0^+)}{v_i(0^+)}$$

$$= \frac{q_o / C_2}{q_o\left[\dfrac{C_1 + C_2}{C1C2}\right]} = \frac{C1}{C1 + C2} \quad \text{—— (1)}$$

(ii) At $t = \infty$, steady state has been reached. The capacitors acts as open circuits.



we have output $v_o(\infty) = iR_2$

$$\text{current } i = \frac{v_i(\infty)}{R_1 + R_2}$$

$$v_o(\infty) = \frac{v_i(\infty)}{R_1 + R_2} \cdot R_2$$

$$\frac{v_o(\infty)}{v_i(\infty)} = \frac{R_2}{R_1 + R_2}$$

here $\frac{v_o(\infty)}{v_i(\infty)}$ = final attenuation

$$\therefore \text{final attenuation} = \frac{R_2}{R_1 + R_2} \quad \text{—— (2)}$$

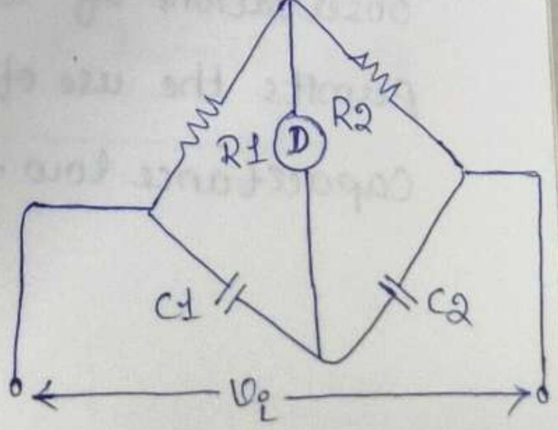
For compensation to be complete, it is essential that initial attenuation = final attenuation.

$$\therefore \frac{C1}{C1 + C_2} = \frac{R_2}{R_1 + R_2} \quad \text{from (1) \& (2)}$$

$$C1\,R1 + C1\,R_2 = C1\,R_2 + C2\,R_2$$

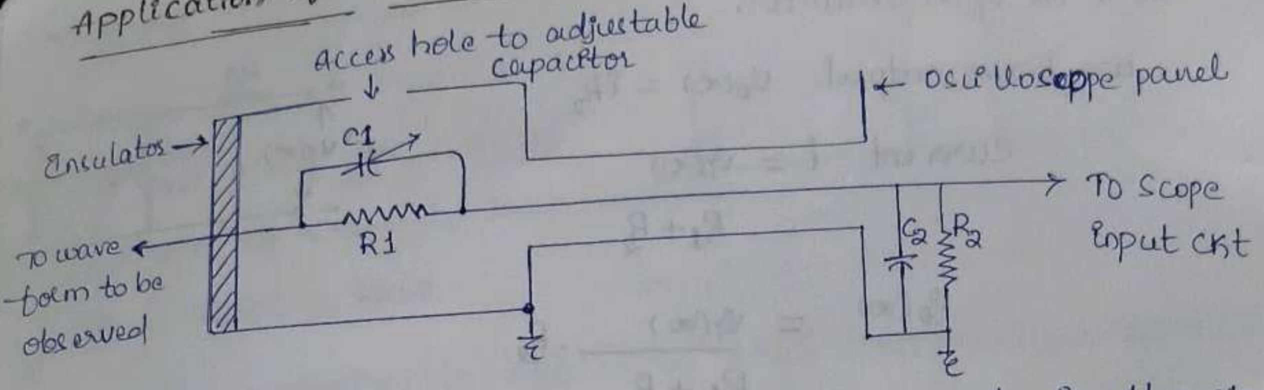
$$\boxed{C1\,R1 = C2\,R_2}$$

This condition enables us to visualise the practical attenuator circuit of figure as a balanced desauty bridge.



D = detector

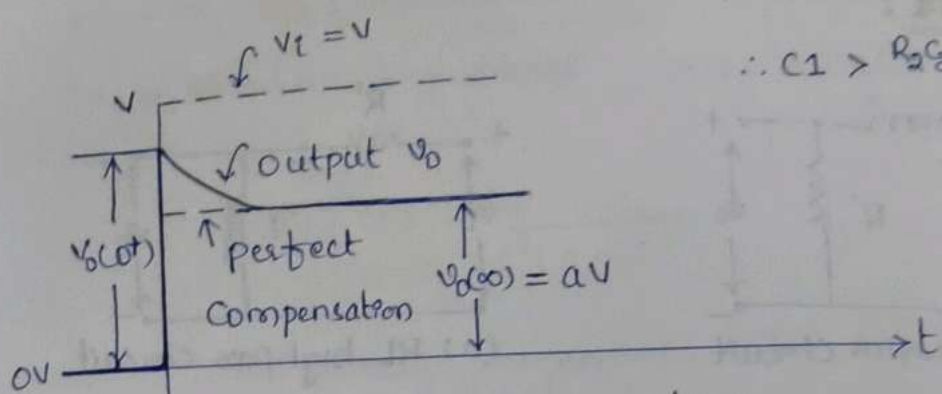
Application of attenuator as a CRO probe :



To measure the signal at a point in the circuit, the input terminals of the oscilloscope are connected to the signal point. Normally the point at which the signal is available will be at some distance from the oscilloscope terminals and if the signal appears at a high impedance level, a shielded cable is used to connect the signal to the oscilloscope. The shielding is neccessary in this case to isolate the input lead from stray fields such as those of the ever-present power line. The capacitance seen looking into several feet of cable may be as high as 100 to 150 pf. This combination of high input capacitance together with the high output impedence of the signal source will make it impossible to make faithful observations of waveforms. A probe assembly, which permits the use of shielded cable and still keeps the capacitance low.

$v_o(0^+) > v_o(\infty)$

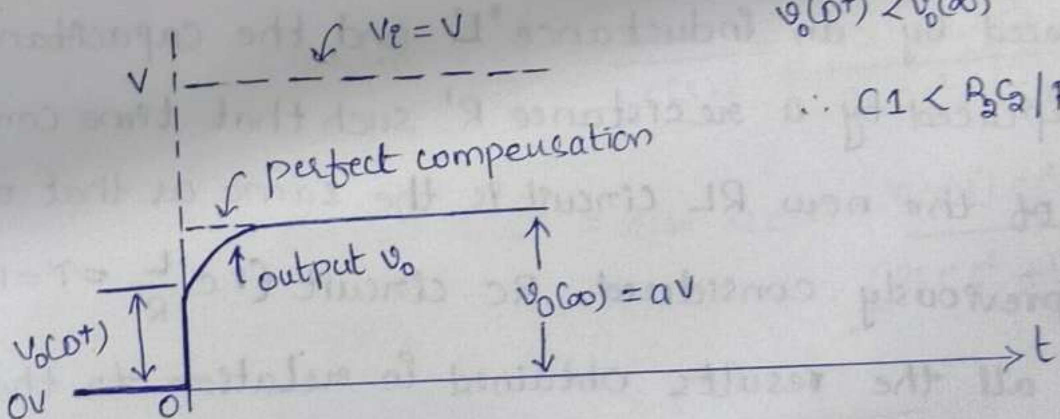
$\therefore C1 > R_2C_2/R1$



(a) Over Compensation

$v_o(0^+) < v_o(\infty)$

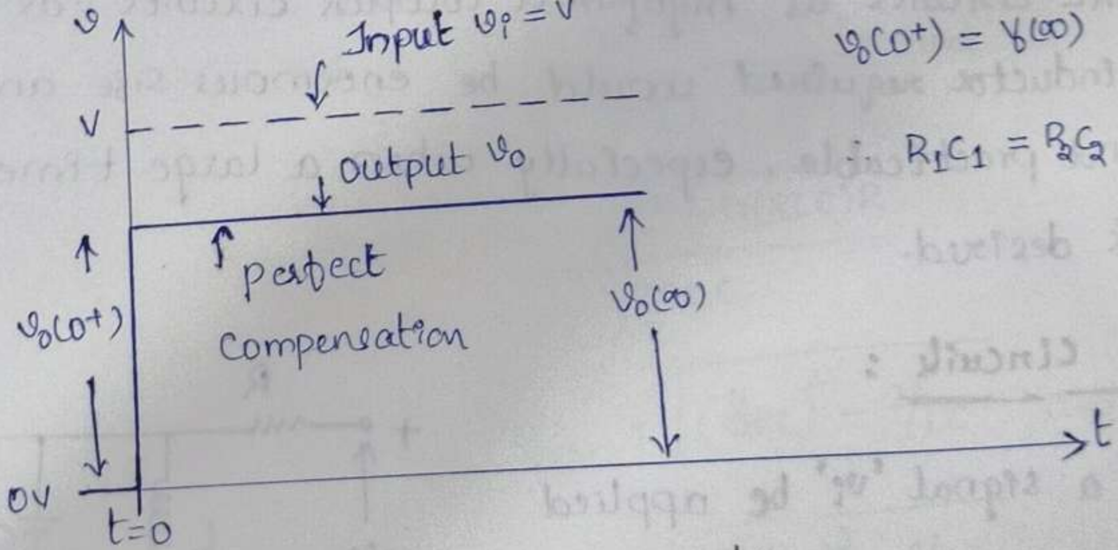
$\therefore C1 < R_2C_2/R1$



(b) Under compensation

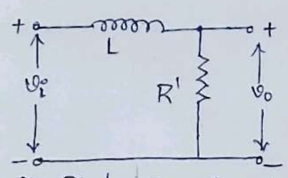
$v_o(0^+) = v_o(\infty)$

$\therefore R_1C_1 = R_2C_2$

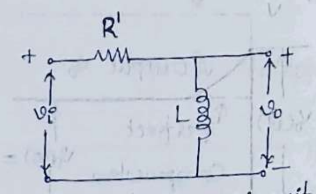


(C) Perfect Compensation

RL Circuits :



(a) RL lowpass circuit



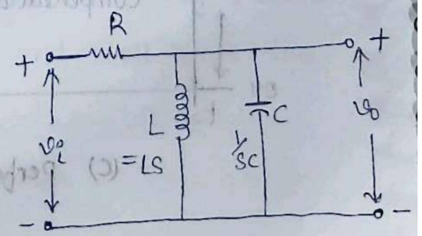
(b) RL highpass circuit.

The time constant of RC lowpass and highpass circuits is given as $\tau = RC$. If the resistance 'R' is replaced by an inductance 'L' and the capacitance 'C' is replaced by a resistance R' such that time constant $\left(\frac{L}{R'}\right)$ of the new RL circuit is the same as that of the previously considered RC circuit (i.e $\frac{L}{R'} = \tau = RC$), then all the results obtained in relation to the RC circuit also hold good for the RL circuit.

however it is not common practise to use RL circuits as highpass or lowpass circuits, as the inductor required would be enormous size and hence not practicable, especially when a large time constant is desired.

RLC circuits :

Let a signal 'v_i' be applied to the LC parallel circuit through resistor 'R'.



Ls and $1/Cs$ impedences are in parallel.

The equivalent impedance $Z_{LC}(s) = \dfrac{Ls(1/Cs)}{Ls + 1/Cs}$

$$Z_{LC}(s) = \frac{Ls}{s^2LC+1}$$

Total circuit impedance $Z(s) = R + \dfrac{Ls}{s^2LC+1}$

$$Z(s) = \frac{RLCs^2+Ls+R}{LCs^2+1}$$

Current $I(s) = \dfrac{V_i(s)}{Z(s)} = \dfrac{V_i(s)\,(LCs^2+1)}{RLCs^2+Ls+R}$

Output $V_o(s) = I(s) \cdot Z_{LC}(s) = \dfrac{V_i(s)(LCs^2+1)}{(RLCs^2+Ls+R)} \times \dfrac{Ls}{LCs^2+1}$

(or) $\dfrac{V_o(s)}{V_i(s)} = \dfrac{Ls}{RLCs^2+Ls+R}$

$RLCs^2+Ls+R=0$, Roots of this quadratic equation are

$$s_1 = \frac{-L+\sqrt{L^2-4(RLC)R}}{2RLC} \text{ and}$$

$$s_2 = \frac{-L-\sqrt{L^2-4(RLC)R}}{2RLC}$$

(or) $s_1 = -1/2RC + \sqrt{(1/2RC)^2 - 1/LC}$ and

$$s_2 = -1/2RC - \sqrt{(1/2RC)^2 - 1/LC}$$

we have the damping constant $\left(\frac{1}{2RC}\right)^2 > \frac{1}{LC}$

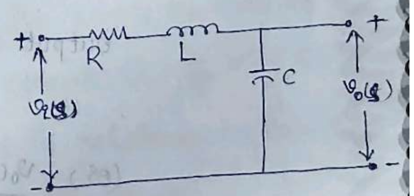
i.e $\frac{1}{2RC} > \frac{1}{\sqrt{LC}} \Rightarrow 2R\sqrt{C}\cancel{\sqrt{C}} > \sqrt{L}\cancel{\sqrt{C}} \Rightarrow R > \frac{1}{2}\sqrt{L/C}$

(i) The circuit is overdamped if, $R < \frac{1}{2}\sqrt{L/C}$

(ii) Critically damped if $R = \frac{1}{2}\sqrt{L/C}$

(iii) Underdamped if $R > \frac{1}{2}\sqrt{L/C}$

RLC Series circuit :



from the circuit

$$v_o(s) = \frac{v_i(s) \cdot 1/sC}{R + Ls + 1/sC}$$

$$\frac{v_o(s)}{v_i(s)} = \frac{1}{sCR + Ls^2C + 1}$$

$$\frac{v_o(s)}{v_i(s)} = \frac{1}{LC\left[s^2 + \frac{R}{L}s + \frac{1}{LC}\right]}$$

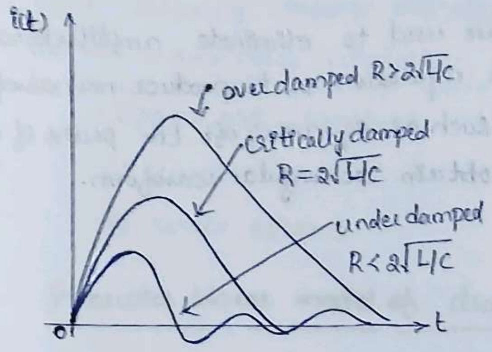
The roots of the characteristic equation $s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$

$$s_1, s_2 = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

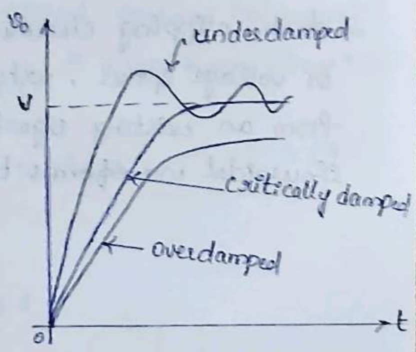
(i) If $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$ i.e $R > 2\sqrt{L/C}$, both the roots are real and different. The circuit is overdamped and there are no oscillations in the output.

(ii) If $(R/2L)^2 = 1/LC$ i.e $R = 2\sqrt{L/C}$, both the roots are real and equal . The circuit is critically damped.

(iii) If $(R/2L)^2 < 1/LC$ i.e $R < 2\sqrt{L/C}$ the roots are complex conjugate of each other . The circuit is underdamped and there will be oscillations in the output.



(a) Current response



(b) Voltage response

Ringing circuit : To obtain a pulse from a step voltage (Peaking) the circuit should operate in the neighbourhood of critical damping . In some applications almost undamped oscillations are required . A circuit , which provides as nearly undamped oscillations as possible is called a "ringing circuit". If the damping is very small the circuit will ring for many cycles. Many times the value of 'Q' of a circuit which has to ring for a given number 'N' of cycles before the amplitude decreases to 1/e of its initial value needs to be known. This is given by $Q = \pi N$.

Thus, a circuit with $Q = 22$ will ring for $Q/\pi = 4$ cycles before the amplitude of the oscillation decreases to 37% of its initial value. A ringing circuit may be used to generate a sequence of pulses regularly spaced in time. These pulses find application in many timing operations.

⇒ Diode clipping circuits are used to eliminate amplitude noise or voltage spikes, voltage regulation or to produce new waveforms from an existing signal such as squaring off the peaks of a sinusoidal waveform to obtain rectangular waveform.